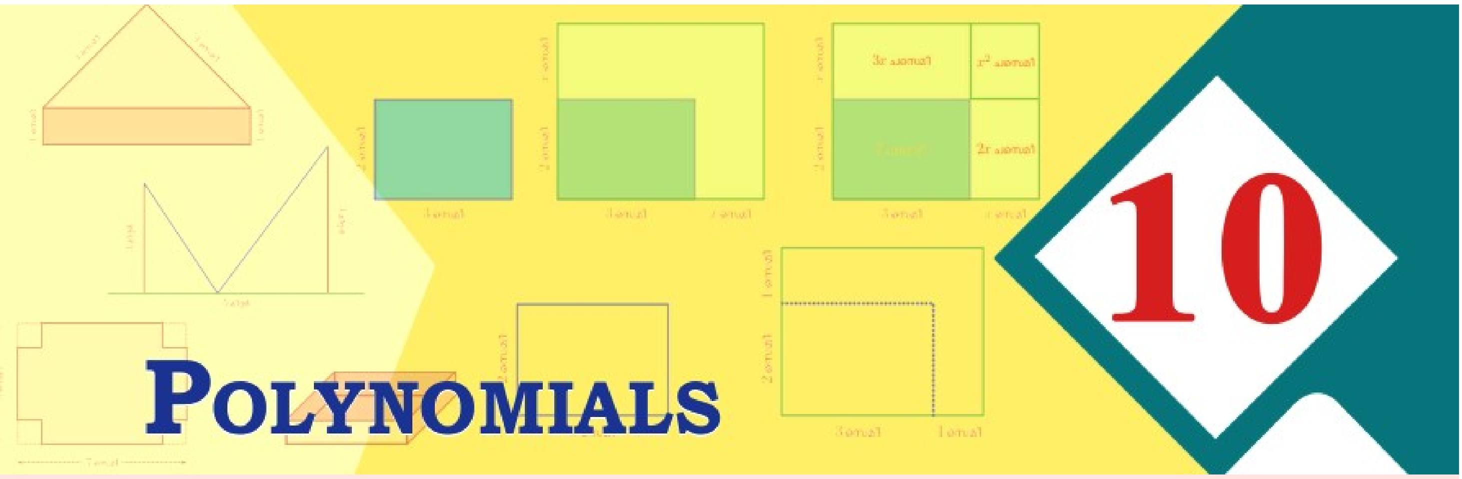

Algebra of measurements

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(1) In all rectangles with one side 1 centimetre less than the other, denote the length of the shorter side as x centimetres

(i) Denote their perimeters as $p(x)$ centimetres and write the relation between x and $p(x)$ as an equation

(ii) Denote their areas as $a(x)$ square centimetres and write the relation between x and $a(x)$ as an equation

(iii) Compute $p(1)$, $p(2)$, $p(3)$, $p(4)$, $p(5)$. Do you see any pattern?

(iv) Compute $a(1)$, $a(2)$, $a(3)$, $a(4)$, $a(5)$. Do you see any pattern?

Answer

Given one side is smaller than the other by 1 cm.

∴ The two adjacent sides of the rectangle are x, x + 1.

(i) perimeter $p(x) = 2 \times [x + (x + 1)]$

$= 2\times [x+x+1] = 2\times(2x+1)$

$\therefore p(x) = 4x + 2$

(ii) Area, $a(x) = (x) \times (x + 1)$

$\therefore a(x) = x^2 + x$

(iii) $p(x) = 4x + 2$

$p(1) = 4 \times 1 + 2 = 6$

$p(2) = 4 \times 2 + 2 = 10$

$p(3) = 4 \times 3 + 2 = 14$

$p(4) = 4 \times 4 + 2 = 18$

$p(5) = 4 \times 5 + 2 = 22$

Here the difference between a number and its successor is always 4.

(iv) $a(x) = x^2 + x$

$a(1) = 1^2 + 1 = 2$

$a(2) = 2^2 + 2 = 6$

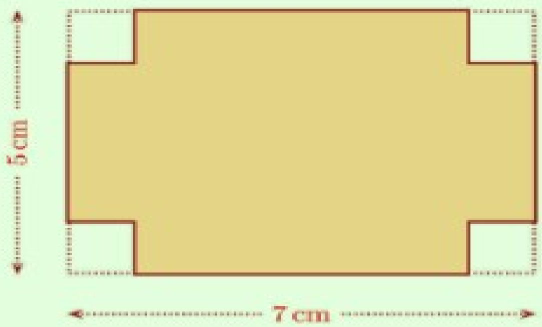
$a(3) = 3^2 + 3 = 12$

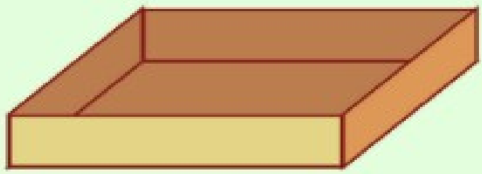
$a(4) = 4^2 + 4 = 20$

$a(5) = 5^2 + 5 = 30$

Here the difference between a number and its previous number is increasing by 2.

(2) From the four corners of a rectangle, small squares of the same size are cut off and the tabs are raised up to make a box as in the picture below:



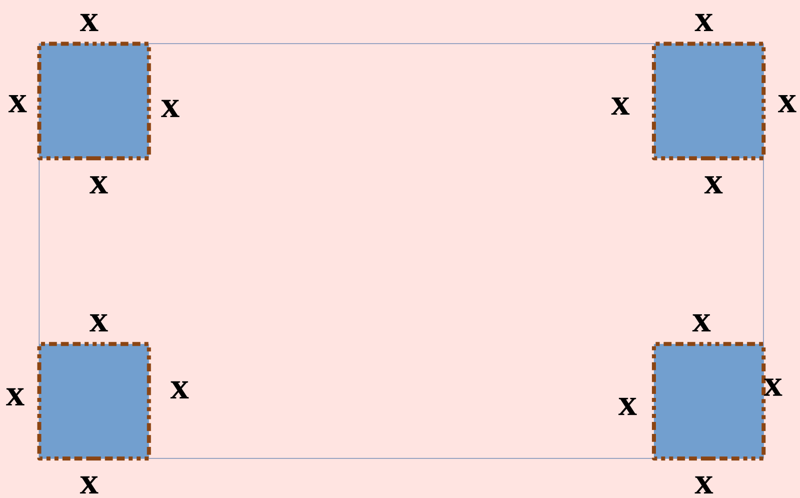


(i) Denote the length of the sides of the squares as x centimetres and write the the lengths of the three edges of the box in terms of x

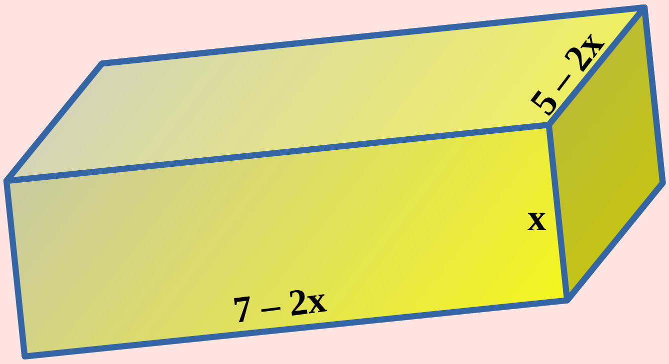
(ii) Denote the volume of the box as $v(x)$ cubic centimetres and write the relation between x and $v(x)$ as an equation

(iii) Compute $v\left(\frac{1}{2}\right)$, $v(1)$ and $v\left(1\frac{1}{2}\right)$

Answer



(i) **The side of the square = x**
The dimensions of the box are
Length = 7 – 2x
Breadth = 5 – 2x
Height = x



(ii) $v(x) = L \times B \times H$

$v(x) = (7 - 2x) \times (5 - 2x) \times x$

$v(x) = (35 - 14x - 10x + 4x^2) \times x$

$v(x) = 4x^3 - 24x^2 + 35x$

(iii) $v\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 24\left(\frac{1}{2}\right)^2 + 35\left(\frac{1}{2}\right)$

$v\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right) - 6 + 35\left(\frac{1}{2}\right) = 30$

$v(1) = 4(1)^3 - 24(1)^2 + 35 \times 1 = 15$

$v\left(1\frac{1}{2}\right) = v\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^3 - 24\left(\frac{3}{2}\right)^2 + 35\left(\frac{3}{2}\right)$

$= \left(\frac{27}{2}\right) - 54 + \left(\frac{105}{2}\right)$

$= 66 - 54 = 12$

(3) Consider all rectangles that can be made with a rope of length 1 metre. Denote the length of one side as x centimetres and the area enclosed by the rope as $a(x)$ square centimetres.

(i) Write the relation between x and $a(x)$ as an equation

(ii) Why are $a(10)$ and $a(40)$ the same number?

(iii) To get the same number as $a(x)$ when x is taken as two different numbers, what should be the relation between the numbers?

Answer

According to the question the perimeter of the rectangle must be always equal to 100 cm.

Now, let one side be 'x' cm and other adjacent side be 'y' cm.

Perimeter $= 2 \times (x + y)$

$100 = 2 \times (x + y)$

$\frac{100}{2} = x+y$

$x+y=50$

$y = 50 - x$

(i) $a(x) = (50-x) \times x$

$a(x) = 50x - x^2$

(ii) $a(10) = 50 \times 10 - 10^2$

$a(10) = 500 - 100 = 400$

$a(40) = 50 \times 40 - 40^2 = 400$

$a(40) = 2000 - 1600 = 400$

Now if a side x is 10 cm, then the other side

$y = 50 - 10$

$y = 40$ cm

∴ In the case of x = 10 cm y = 40 cm, and in case of x = 40 cm y = 10 cm.

So, in both cases we have same rectangle hence the area is same.

Special expressions

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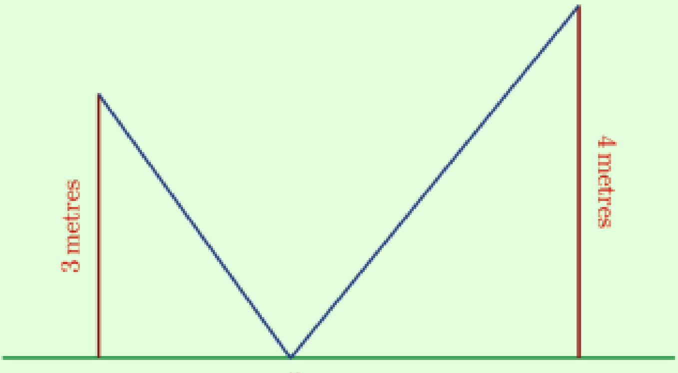
(1) In each of the problems below, check whether the relation between the specified measurements is a polynomial. Give reasons for your assertions.

(i) The relation between the length of the sides of a square park and the area of a 1 metre wide path around it.

(ii) The relation between the amount of acid added to a mixture of 3 litres of acid and 7 litres of water, and the change in the percent of acid in the mixture

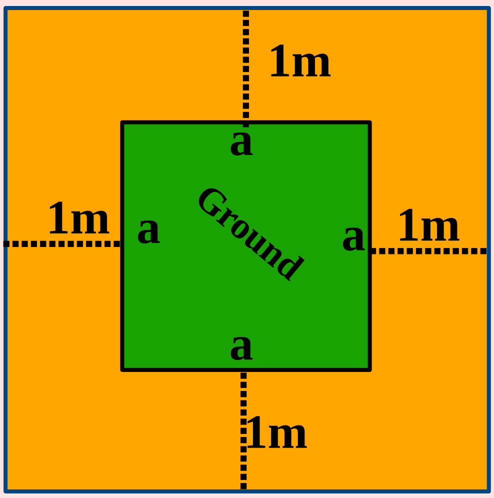
(iii) Two poles of heights 3 metres and 4 metres stand 5 metres apart. A rope is to be stretched from the top of one post to some point on the ground and then stretched to the top of the other pole:



The distance from the foot of one pole to the point on the ground where the rope is fixed, and the total length of the rope.

Answer

(i)



Let the side of the ground be ‘a’. The width of the path is 1 m.

Length of one side of the path is (a + 2) m.

Area of the path = $(a + 2)^2 – a^2$
= $a^2+4a+4 – a^2$
= 4a+4

Now, Area of the path is ‘a’ is defined p(a)

p(a) = 4a+4

(ii) A liquid is 7 litres water and 18 litres acid.

Let ‘x’ litres acid is added to this liquid.

Now, The liquid is 7 litres water and (18 + x) litres acid.

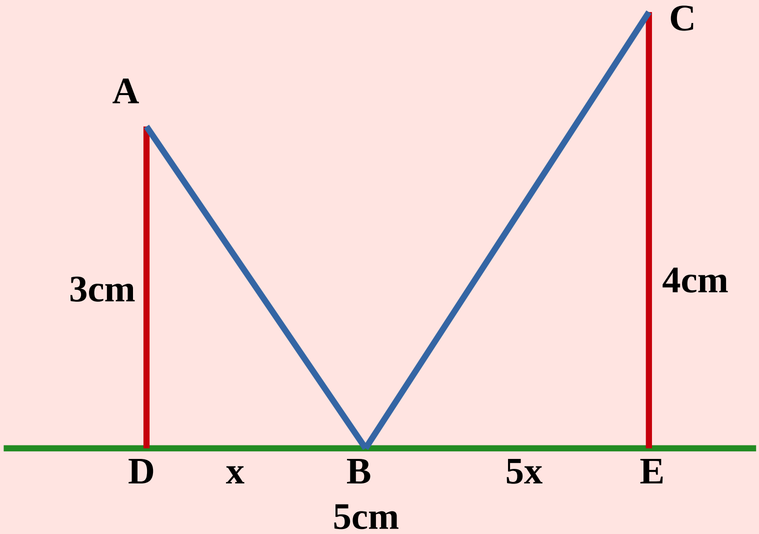
Initial percentage of acid = $\frac{18}{18+7} \times 100 = \frac{18}{25} \times 100 = 72\%$

Initial percentage of acid = 72%

Final percentage of acid = $\frac{18+x}{18+x+7} \times 100$

Change in percentage of the acid = ($\frac{18+x}{18+x+7} \times 100$) - 72

(iii)



Let the distance of the point on ground be x m from the 3 m pole.

∴ The distance of the point on ground from 4 m pole is (5-x) m

The length of the rope is AB + BC.

By Pythagoras Theorem

$AB^2 = AD^2 + DB^2$

$AB^2 = 3^2 + x^2$

$AB = \sqrt{9+x^2}$

Similarly, $BC = \sqrt{16+x^2}$

∴ The length of rope = $\sqrt{9+x^2} + \sqrt{16+x^2}$

(2) Write each of the following operations as an algebraic expression. Check which of them are polynomials, giving reasons

(i) Sum of a number and its reciprocal

(ii) Sum of a number and its square root

(iii) The product of the sum of a number and its square root, and the difference of the square root and the number

Answer

(i) **Let the number be x.**

So it's reciprocal is $\frac{1}{x}$

Sum = $x + \frac{1}{x} = x + x^{-1}$

It is not a polynomial as it has a negative power of x.

(ii) **(ii) Let the number be x.**

So it's square root is $\sqrt{x}$

Sum = $x + \sqrt{x}$

It is not a polynomial as it has square root which is fractional exponent

(iii) The sum of number and it's square root = $x + \sqrt{x}$

The difference of number and it's square root = $x - \sqrt{x}$

Their Product = $(x + \sqrt{x}) \times (x - \sqrt{x}) = x^2 - x$

$\therefore$ It is a Polynomial as all the exponents of x are positive integer.

(3) For each of the polynomial $p(x)$ given below, compute $p(1)$ and $p(10)$

(i) $p(x) = 2x + 5$ (ii) $p(x) = 3x^2 + 6x + 1$ (iii) $p(x) = 4x^3 + 2x^2 + 3x + 7$

Answer

(i) $p(x) = 2x + 5$

$p(1) = 2 \times 1 + 5$

$p(1) = 2 + 5 = 7$

$p(10) = 2 \times 10 + 5$

$p(10) = 20 + 5 = 25$

(ii) $p(x) = 3x^2 + 6x + 1$

$p(1) = 3 \times 1^2 + 6 \times 1 + 1$

$p(1) = 3+6+1 = 10$

$p(10) = 3 \times 10^2 + 6 \times 10 + 1$

$p(10) = 3 \times 100 + 60 + 1 = 300+60+1 = 361$

(iii) $p(x) = 4x^3 + 2x^2 + 3x + 7$

$p(1) = 4 \times 1^3 + 2 \times 1^2 + 3 \times 1 + 7$

$p(1) = 4+2+3+7 = 16$

$p(10) = 4 \times 10^3 + 2 \times 10^2 + 3 \times 10 + 7$

$p(10) = 4000+200+30+7 = 4237$

(4) For each of the polynomial $p(x)$ given below, compute $p(0)$, $p(1)$ and $p(-1)$

(i) $p(x) = 3x + 5$ (ii) $p(x) = 5x - 8$ (iii) $p(x) = 3x^2 + 6x + 1$

(iv) $p(x) = 2x^2 - 5x + 3$ (v) $p(x) = 4x^3 + 2x^2 + 3x + 7$

(vi) $p(x) = ax^3 + bx^2 + cx + d$

Answer

(i) $p(x) = 3x + 5$

$p(0)= 3\times0 +5 =5$

$p(1)= 3\times1 +5 = 3+5=8$

$p(-1)= 3\times(-1) +5 = -3 + 5 = 2$

(ii) $p(x) = 5x - 8$

$p(0)= - 8$

$p(1)= 5\times1 - 8 = 5-8 = -3$

$p(-1)= 5\times(-1) -8 = -5 - 8 = -13$

(iii) $p(x) = 3x^2 + 6x + 1$

$p(0)= 1$

$p(1) = 3\times1^2+ 6 \times1 + 1$

$p(1)= 3+6+1= 10$

$p(-1) = 3 \times(-1)^2 + 6\times(-1) +1$

$p(-1)= 3+(-6)+1= -2$

(iv) $p(x) = 2x^2 - 5x + 3$

$p(0)= 3$

$p(1) = 2 \times 1^2 - 5 \times 1 + 3$

$p(1) = 2 - 5 + 3 = 0$

$p(-1) = 2 \times -(1)^2 - 5 \times (-1) + 3$

$p(-1) = 2 +5 +3 = 10$

(v) $p(x) = 4x^3 + 2x^2 + 3x + 7$

$p(0) = 7$

$p(1) = 4 \times 1^3 + 2 \times 1^2 + 3 \times 1 + 7$

$p(1) = 4+2+3+7 = 16$

$p(-1) = 4 \times (-1)^3 + 2 \times (-1)^2 + 3 \times (-1) + 7$

$p(-1) = -4 + 2 - 3 + 7 = 2$

(vi) $p(x) = ax^3 + bx^2 + cx + d$

$p(0) = d$

$p(1) = a \times 1^3 + b \times 1^2 + c \times 1 + d$

$p(1) = a + b + c + d$

$p(-1) = a \times (-1)^3 + b \times (-1)^2 + c \times (-1) + d$

$p(-1) = -a + b - c + d$