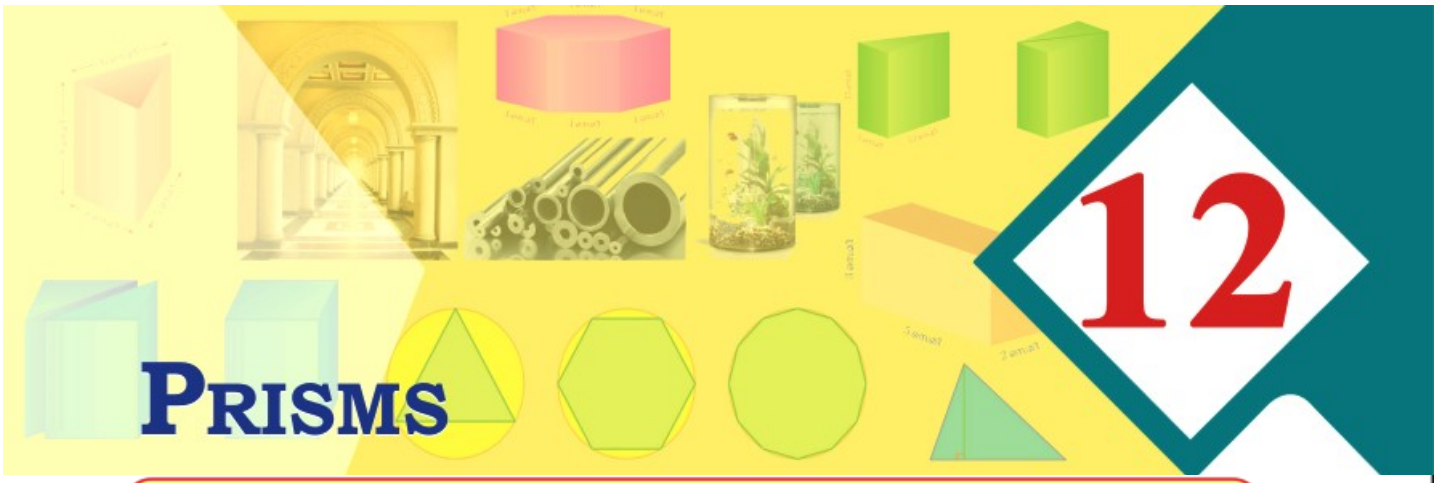

The volume of any prism is the product of the base area and the height.

Volume

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(1) The base of a prism is an equilateral triangle of perimeter 18 centimetres and its height is 5 centimetres. Calculate its volume.

Answer

Perimeter of an equilateral triangle= 18 cm

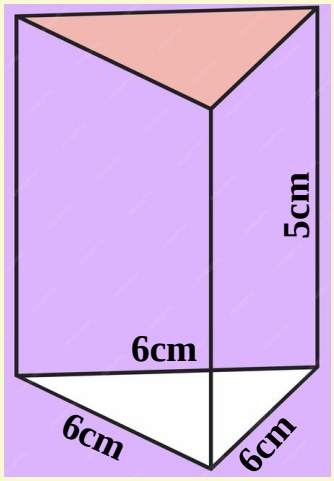
One side , a = $\frac{18}{3}$ **= 6 cm**

Base area = $\frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3}\times 6^2}{4} = 9\sqrt{3}$ **cm²**

Height of the prism = 5cm

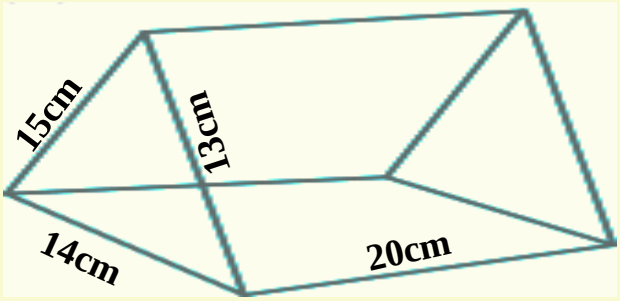
Volume of the prism= Base area × Height

∴ Volume of the prism= $9\sqrt{3} \times 5 = 45\sqrt{3}$ **cm²**



(2) The base of a prism is a triangle of sides 13 centimetres, 14 centimetres and 15 centimetres, and its height is 20 centimetres. Calculate its volume.

Answer



Using Heron's formula, the area of the base = $\sqrt{s(s-a)(s-b)(s-c)}$

s = $\frac{a+b+c}{2} = \frac{13+14+15}{2} = \frac{42}{2} = 21\,cm$

a = 13 cm, b = 14cm, c = 15cm
s - a = 21 - 13 = 8
s - b = 21 - 14 = 7
s - c = 21 - 15 = 6

Base area= $\sqrt{s(s-a)(s-b)(s-c)}$

Base area= $\sqrt{21\times8\times7\times6}$

Base area= $\sqrt{7\times3\times2\times2\times2\times7\times3\times2}$
Base area= 7× 3 × 2 × 2 = 84 cm^2

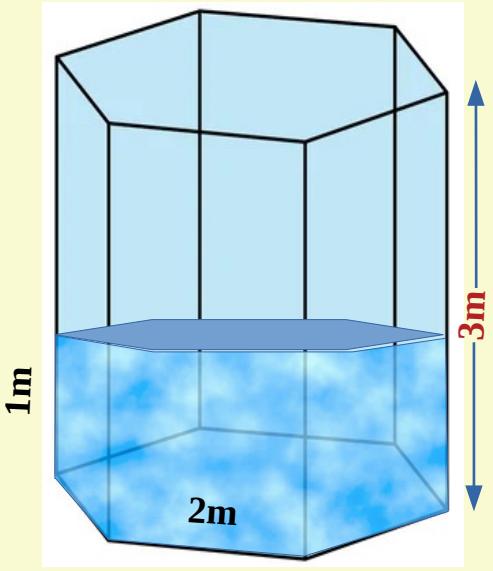
Height of the prism = 20cm
∴ Volume of the prism = Base area × Height
= 84 × 20 = 1680 cm^3

(3) There is a hexagonal pit in the school ground to collect rain water. Each side of the hexagon is 2 metres and the depth of the pit is 3 metres. It now contains water one metre deep. How much litres is this? How much more litres can it contain?

Answer

One side of a hexagon= 2m

Area of the base = Area of the hexagon
= $6 \times \frac{\sqrt{3}a^2}{4}$
= $6 \times \frac{\sqrt{3}\times2^2}{4} = 6\sqrt{3}$ **m^2**
Since there is water at a height of 1 m,
Volume = 6 √ 3 × 1
= 6 √ 3 m^3
= 6 √ 3 × 1000 litre
= 6 × 1.73 × 1000 litre
= 10380 litre



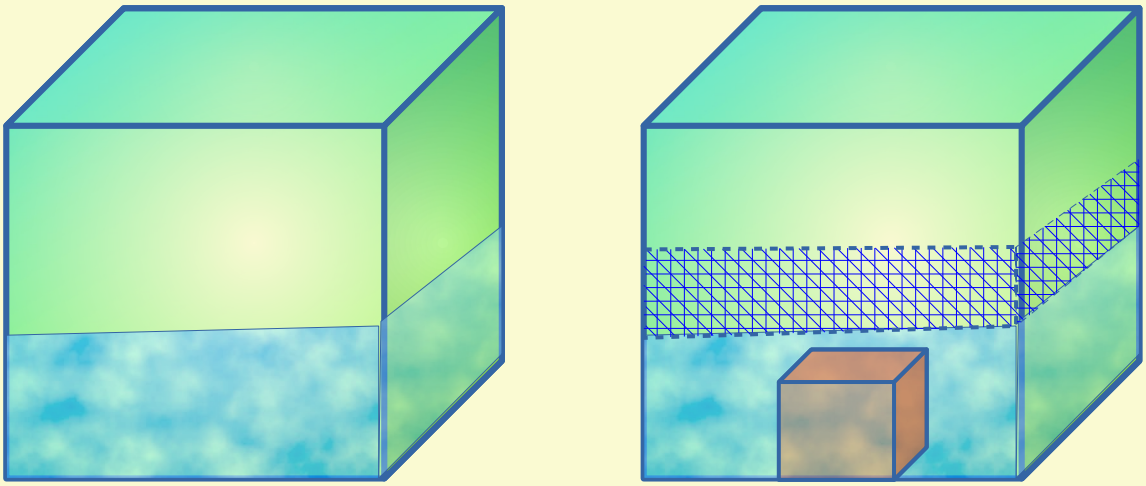
Area of the remaining part = Area of the base × Height of the remaining part
= 6 √ 3 × 2 = 12√ 3m^3

(Total height=3 m , Height of the remaining part = 3-1=2m)

∴ The volume of the remaining part = 12 × 1.73 × 1000 litre = 20760 litre

(4) A hollow prism with base a square of sides 16 centimetres contains water 10 centimetres high. If a cube of edges 8 centimetres is immersed in it, by how much would the water level rise?

Answer



The volume of the rising water is equal to the volume of the cube.

Volume of a cube = Base area × height
Volume of a cube = 8 ×8 ×8

The volume of rising water= 16 × 16 × h

Since the volume of the rising water is equal to the volume of the cube,
16 × 16 × h = 8 ×8 ×8

h= $\frac{8\times8\times8}{16\times16}$ **= 2cm**

∴ **The water level will rise by 2cm.**

(5) A rectangular block of metal has edges of lengths 6 centimetres, 9 centimetres and 15 centimetres. It is melted and recast into identical cubes of sides 3 centimetres. How many cubes would be got?

Answer

Volume of a rectangular block made of metal= 6 × 9 × 15

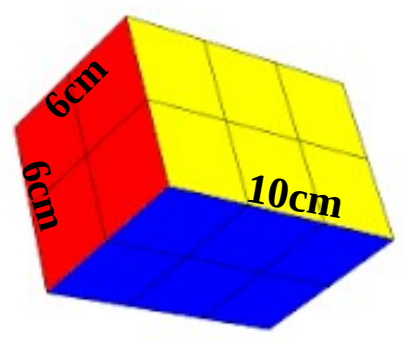
Volume of the cube = 3× 3 × 3

∴ **Number of cubes =** $\frac{6\times9\times15}{3\times3\times3}$ **= 30 number**

(6) The base of a prism is a square of sides 6 centimetres and its height is 10 centimetres. What is its volume? What is the maximum volume of a triangular prism cut from it?

Answer

Volume of a square prism= 6 × 6 × 10 = 360cm^3



Triangular prisms with maximum volume can be cut in different ways.

Volume of the triangular prism = $\frac{1}{2}$ ×10×6×6 = $\frac{1}{2}$ ×6×10×6= $\frac{1}{2}$ ×6× 6×10

= 180cm^3

Area

The lateral surface area of any prism is the product of its base perimeter and height.

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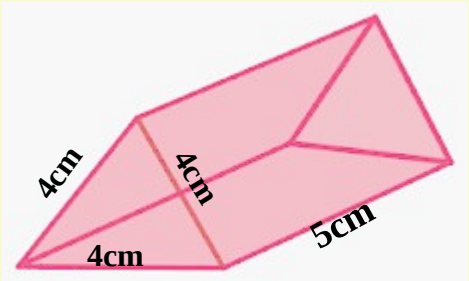
(1) The base of a prism is an equilateral triangle of perimeter 12 centimetres and its height is 5 centimetres. What is its total surface area?

Answer

Lateral surface area = base perimeter × height = 12 × 5 = 60 cm^2

Since the base is an equilateral triangle,
Base perimeter= 12cm

One side = $\frac{12}{3} = 4\,cm$



$\therefore$ **Base area** $= \frac{\sqrt{3}\times 4^2}{4}$

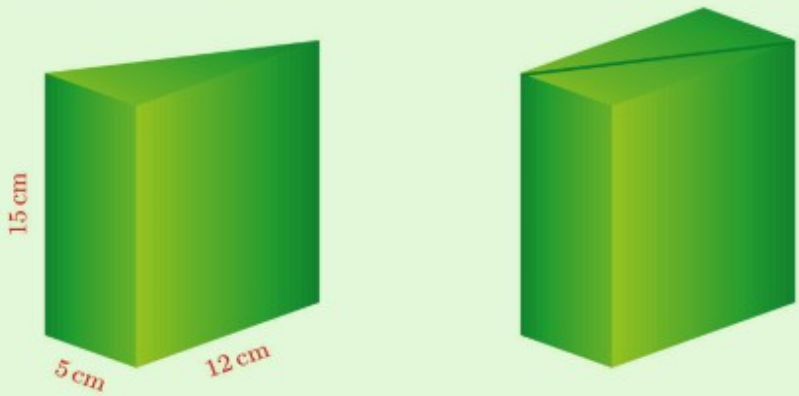
$= \frac{\sqrt{3}\times 16}{4} = 4\sqrt{3}\,cm^2$

Two base area = 2 × $4\sqrt{3}$ **=** $8\sqrt{3}$ **cm^2**

$\therefore$ **Total surface area = Lateral surface area + 2 × base area**

= $60 + 8\sqrt{3}$ **cm^2**

(2) Two identical prisms with right triangles as bases are joined to form a rectangular prism, as in the picture below:



What is the total surface area of this rectangular prism?

Answer

The length , breadth and height of a base of a rectangular prism are 12cm, 5cm and 15cm respectively (Base is a Rectangle)

Lateral surface area of the prism = base perimeter × height
= {2 ×(length + breadth) } × height
= {2 × (12 + 5) } × 15
= 34 × 15= 510 cm^2

Base area = 12 × 5 = 60 cm^2
Total surface area = 510 + 2 × 60
∴ Total surface area= 510 + 120 = 630cm^2

(3) The base of a prism is a triangle with sides 4 centimetres, 13 centimetres and 15 centimetres and its height is 25 centimetres. Calculate its lateral surface area and total surface area.

Answer

The bases of the triangular prism are 4cm, 13cm, and 15cm.

Base perimeter = 4 + 13 + 15 = 32cm

height = 25cm

Lateral surface area = 32 × 25 = 800 cm^2

Base area = $\sqrt{s(s-a)(s-b)(s-c)}$

s = $\frac{a+b+c}{2} = \frac{13+4+15}{2} = \frac{32}{2} = 16\,cm$

a = 13 cm, b = 4cm, c = 15cm
s - a = 16 - 13 = 3
s - b = 16 - 4 = 12
s - c = 16 - 15 = 1

Base area= $\sqrt{16\times3\times12\times1}$

Base area= $\sqrt{16\times4\times3\times3\times1}$ **= 4 × 2 × 3 × 1 = 24cm^2**

∴ Total surface area = 800 + (2×24) = 800 + 48 = 848cm^2

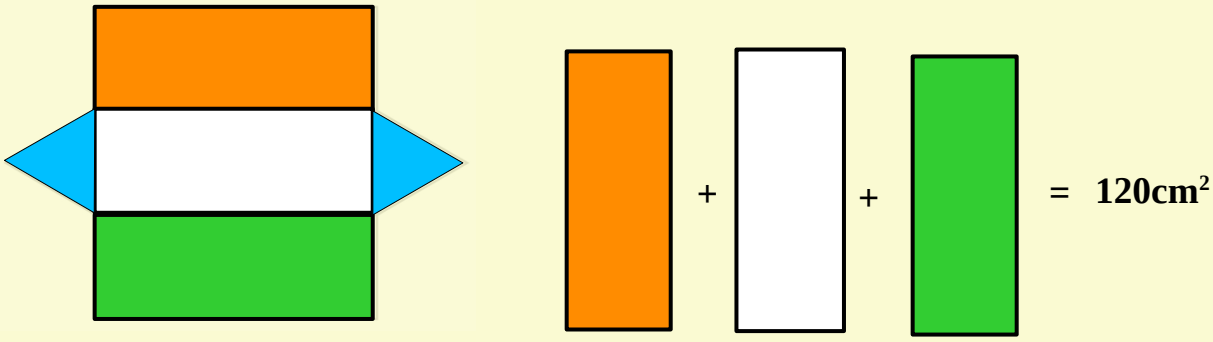
(4) The lateral surface area of a prism, with base an equilateral triangle, is 120 square centimetres

(i) What is the lateral surface area of a prism, with base a rhombus, made by joining two such triangular prisms?

(ii) What is the lateral surface area of a prism, with base an isosceles trapezium, made by joining three such triangular prisms?

(iii) What is the lateral surface area of a prism, with base a regular hexagon, made by joining six such triangular prisms?

Answer



An equilateral triangular prism has 3 equal lateral surface area.

One lateral surface area = $\frac{120}{3} = 40\,cm^2$

i) The lateral surface area of a prism, with base a rhombus, made by joining two such triangular prisms = 40 × 4 = 160 cm² (It has 4 lateral faces)

ii) The lateral surface area of a prism, with base an isosceles trapezium, made by joining three such triangular prisms= 40 × 5 = 200 cm² (It has 5 lateral faces)

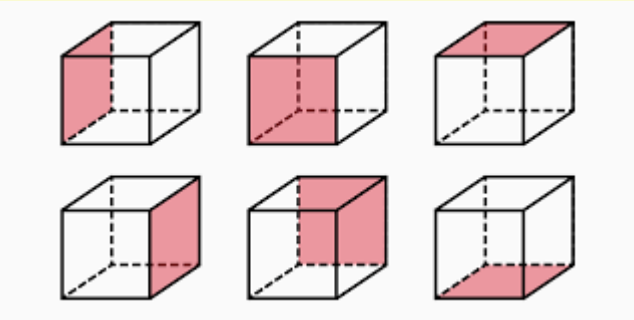
iii) The lateral surface area of a prism, with base a regular hexagon, made by joining six such triangular prisms=40 × 6 = 240cm² (It has 6 lateral faces)

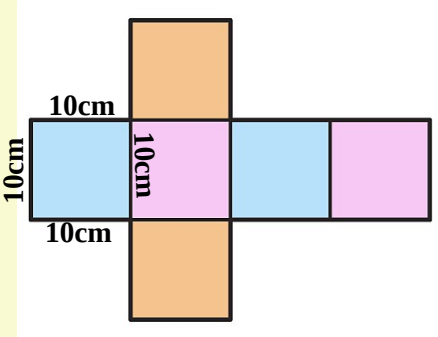
(5) Six sheets of metal, each a square of sides 10 centimetres are joined to make a cube

(i) What is its total surface area?

(ii) How much water can it contain?

Answer



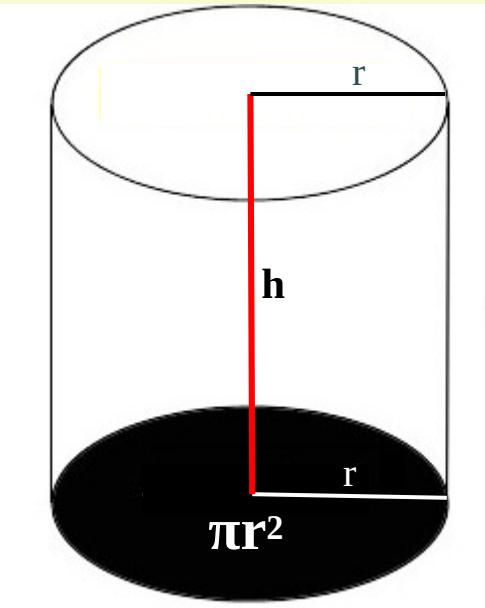


i) **Total surface area = $6 \times 10^2 = 6 \times 100 = 600cm^2$**
(The areas of 6 squares are equal)

ii) **Volume = Base area × height**
= $10 \times 10 \times 10 = 1000\ cm^3$
= $\frac{1000}{1000}$ litre = 1 litre

∴ It contain 1 litre of water .

Cylinders



The volume of a cylinder is the product of the base area and height.

Volume of Cylinder = base area × height

Volume of Cylinder = $\pi r^2 h$

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(1) The base diameter of a cylindrical tank is 1 metre and its height is 2 metres. How many litres of water can it contain?

Answer

Diameter = 1 m

radius = $\frac{1}{2}$ m

height = 2 m

Volume of a cylindrical water tank = $\pi r^2 h$

$$= 3.14 \times \left(\frac{1}{2}\right)^2 \times 2$$

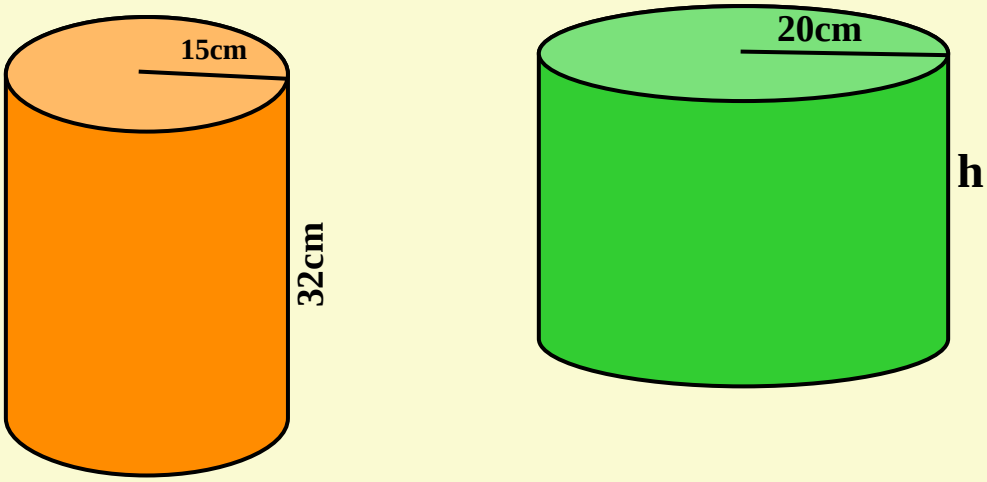
$$= 3.14 \times \frac{1}{2} \times \frac{1}{2} \times 2$$

$$= \frac{3.14}{2} = 1.57 m^3$$

$\therefore$ **Amount of water = 1.57 × 1000 = 1570 litre.**

(2) The base radius of an iron cylinder is 15 centimetres and its height is 32 centimetres, It is melted and recast into a cylinder of base radius 20 centimetres. What is the height of this cylinder?

Answer



The volume of the first cylinder = $\pi r^2 h$

$= \pi \times 15^2 \times 32$

$= \pi \times 15 \times 15 \times 32$

If the height of the cylinder made of iron is taken as 'h',

Volume = $\pi \times 20^2 \times h$ (radius = 20cm)

$= \pi \times 20 \times 20 \times h$

Since the volume of these two cylinders is equal,

$\pi \times 20 \times 20 \times h = \pi \times 15 \times 15 \times 32$

$$h = \frac{\pi \times 15 \times 15 \times 32}{\pi \times 20 \times 20}$$

$\therefore$ **height , h = 18cm**

(3) The base radii of two cylinders of the same height are in the ratio 3 : 4. What is the ratio of their volumes?

Answer

If the radii are '3r' and '4r' and the height is 'h'.

Ratio of their volumes = π × 3r × 3r × h : π × 4r × 4r × h

$$= \frac{\pi \times 3r \times 3r \times h}{\pi \times 4r \times 4r \times h}$$

$$= \frac{9}{16}$$

∴ **Ratio of their volumes = 9 : 16**

(4) The base radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5: 4

(i) What is the ratio of their volumes?

(ii) The volume of the smaller cylinder is 720 cubic centimetres. What is the volume of the larger one?

Answer

i) If the radii are 3r and 4r and the heights are 5h and 4h, then

Ratio of their volumes = π × $(2r)^2$ × 5h : π × $(3r)^2$ × 4h

$$= \frac{\pi \times 2r \times 2r \times 5h}{\pi \times 3r \times 3r \times 4h} = \frac{5}{9}$$

∴ **Ratio of their volumes = 5 : 9**

ii) If the volume of the large cylinder is taken as V, then
720 : V = 5 : 9

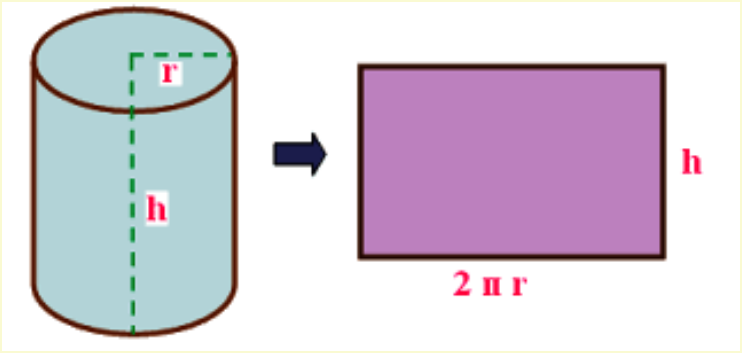
720 × 9 = 5V

$$V = \frac{720 \times 9}{5} = 1296 \text{ cm}^3$$

∴ **The volume of the large cylinder = 1296 cm^3**

Curved surface

The curved surface area of a cylinder is the product of its base circumference and its height



Curved surface area = base perimeter × heiight

Curved surface area = 2πrh

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(1) In a school building there are 12 cylindrical pillars, each of base diameter $\frac{1}{2}$ metre and height 4 metres.

(i) What is the curved surface area of a pillar?

(ii) What is the total cost of painting all the pillars at 80 rupees per square metre?

Answer

The radius of the pillar $= \frac{1}{2} \div 2$

$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ **m**

Height of pillar = 4m

i) **Curved surface area of a pillar = 2πrh**

$= 2 \times \pi \times \frac{1}{4} \times 4$

$= 2\pi\ m^2$

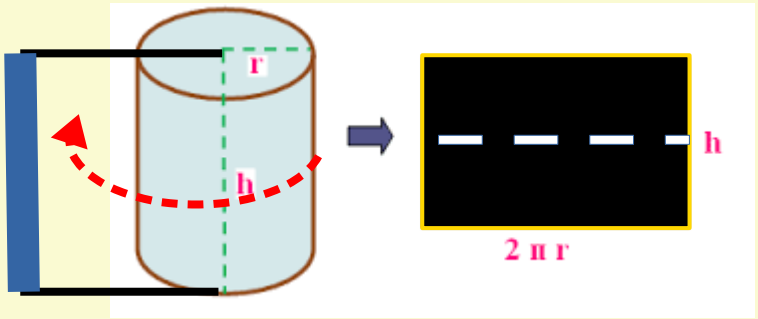
ii) **The total cost of painting 12 pillars at a rate of ₹ 80 per m^2 is = 2 π × 80 × 12**

= 2 × 3.14 × 80 × 12 = ₹ 6028.80

(2) The drum of a road roller has base diameter 80 centimetres and length 1.2 metres:

What is the area of the road levelled when it rolls once?

Answer



The radius of the roller $= \frac{80}{2}$ **= 40cm**

Height of roller = 1.20m = 120cm

Area of the leveled area when the roller rotates once =

Area of the curved surface of the cylinder

= 2 π rh

= 2 × 3.14 × 40 × 120

= 30144 cm^2 = 3.0144m^2

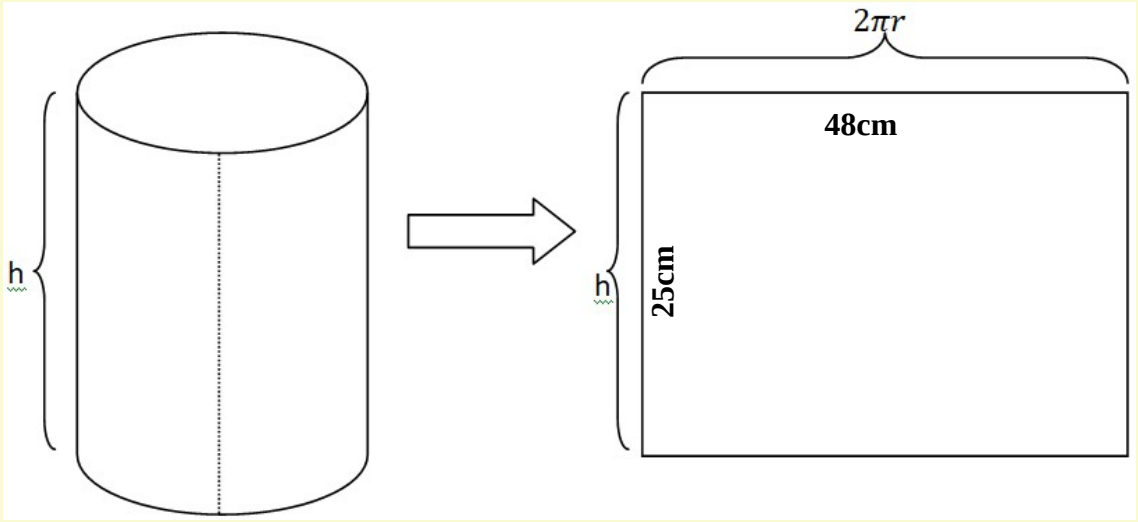
(3) The curved surface area of a cylinder is equal to its base area. What is the relation between its base radius and height?

Answer

Base are of cylinder = πr^2
Curved surface area = $2\pi rh$
If both are equal
$\pi r^2 = 2\pi rh$
$r = 2h$
The radius of a cylinder is twice the height of the cylinder.

(4) A rectangular sheet of metal with sides 48 centimetres and 25 centimetres is rolled into a cylinder of height 25 centimetres and its ends are closed with exactly fitting circles. What is the total surface area of this cylinder?

Answer



Base perimeter of the cylinder = $2\pi r = 48$cm (length = 48cm)

radius of a cylinder, $r = \frac{48}{2\pi} = \frac{24}{\pi}$ cm

height, h = 25cm (breadth =25cm)

Total surface area $= 2\pi r^2 + 2\pi rh$

$$= 2\pi r\,(r + h)$$

$$= 2 \times \pi \times \frac{24}{\pi}\left(\frac{24}{\pi} + 25\right)$$

$$= \frac{48 \times 24}{\pi} + 48 \times 25$$

$\therefore$ **Total surface area** $= \frac{1152}{\pi} + 1200\ \text{cm}^2$