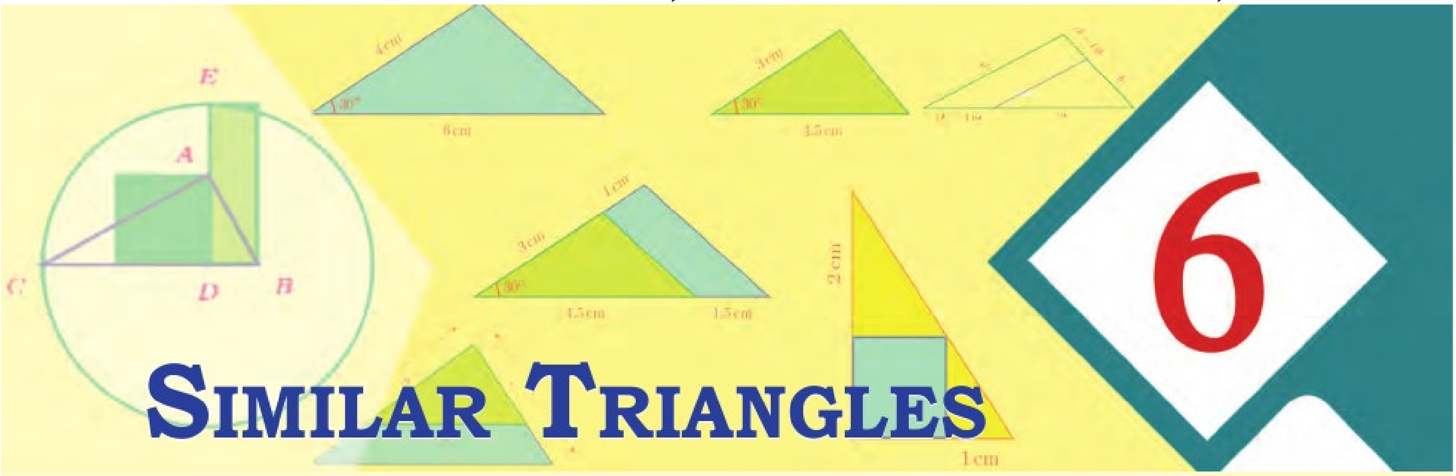


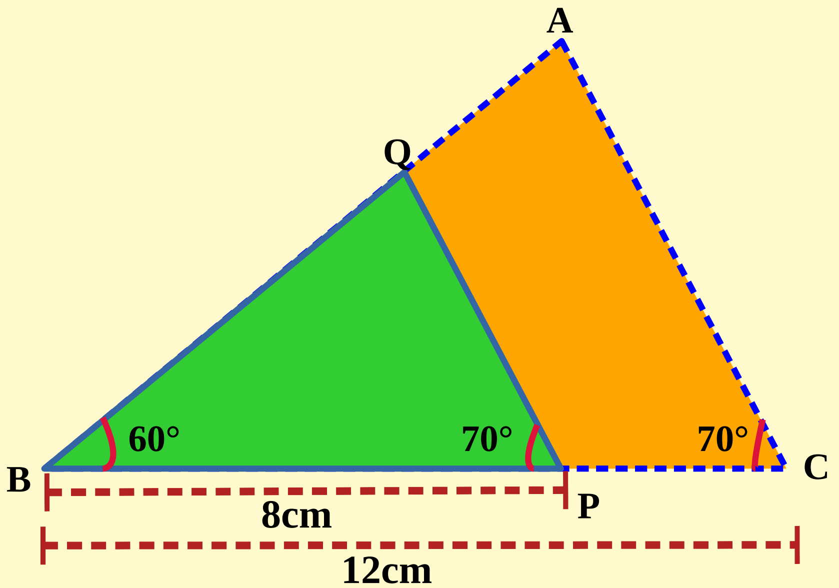
In triangles with the same angles, the sides, in the order of lengths, are in the same ratio.

In triangles with the same angles, the sides opposite equal angles are scaled by the same factor.

Page No: 105, 106 & 107

(1) One side of a triangle is 8 centimetres and the two angles on it are 60° and 70°. Draw the triangle with lengths of sides $1\frac{1}{2}$ times that of this triangle and with the same angles.

Answer



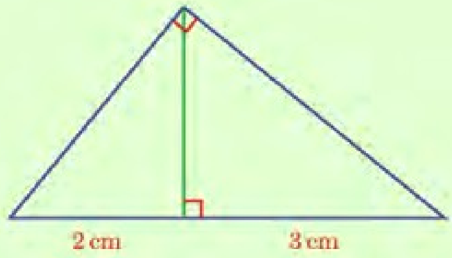
Since BP=8cm, take one and a half times of this,BC=12cm and after drawing the line, take 60° and 70° from B and C, then complete the triangle ABC.

(2) In a right triangle, the perpendicular from the square corner to the hypotenuse divides it into pieces 2 centimetres and 3 centimetres long:

(i) Prove that the two small right triangles formed by this perpendicular have the same angles

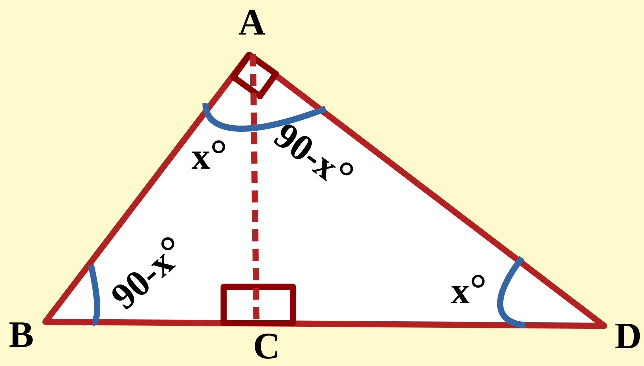
(ii) Taking the height of the perpendicular as h, prove that $\frac{h}{2} = \frac{3}{h}$.

(iii) Calculate the lengths of the perpendicular sides of the original large triangle.



(iv) Prove that if the length of the perpendicular from the square corner of a right triangle to the hypotenuse is h and it divides the hypotenuse into pieces of length a and b, then $h^2 = ab$.

Answer



(i) **Δ**ABC and **Δ**ADC among these

If **Δ**ABC is considered,

If ∠D = x°

∠DAC = 90 –x°, ∠ACD = 90°

If **Δ**ADC is considered ,

∠CAB = x°

∴ ∠B = 90 –x°

∴ The angles in **Δ**ABC and **Δ**ADC are equal.

(ii) Since the angles in **Δ**ABC and **Δ**ADC are equal, the length of the sides is the same ratio

$$\frac{AC}{BC} = \frac{CD}{AC}$$

$$\frac{h}{2} = \frac{3}{h}$$

(iii) from $\frac{h}{2}=\frac{3}{h}$

$h^2 = 6$

In ΔABC

$AB^2 = h^2 + 2^2$

$AB^2 = 6 + 4$

$AB^2 = 10$ cm

AB = $\sqrt{10}$ cm

In ΔADC

$AD^2 = h^2 + 3^2$

$AD^2 = 6 + 9$

$AD^2 = 15$ cm

AD = $\sqrt{15}$ cm

∴ perpendicular sides $\sqrt{10}$ cm, $\sqrt{15}$ cm

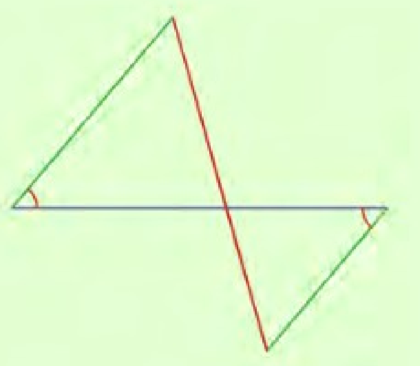
(iv) In $\frac{h}{2}=\frac{3}{h}$

If a and b are taken instead of 2 and 3

$\frac{h}{a}=\frac{b}{h}$

We get h^2= ab .

(3) At the two ends of a horizontal line, angles of the same size are drawn and two points on these slanted lines are joined :

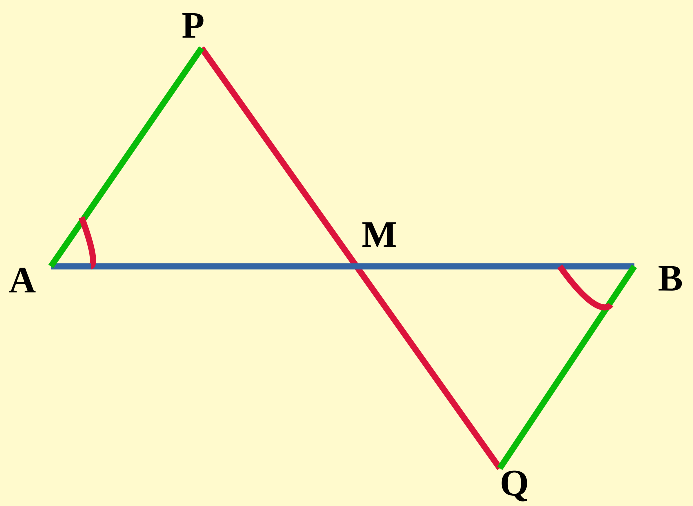


(i) Prove that the horizontal line (blue) and the slanted line (red) cut each other into parts in the same ratio.

(ii) Prove that the slanted lines (green) at the ends of the horizontal line are also in the same ratio.

(iii) Explain how this idea can be used to divide a 6 centimetre long line in the ratio 3 : 4.

Answer



i) In ΔAPM , ΔBQM

∠A = ∠B (given)

∠AMP = ∠BMQ (opposite angles are equal)

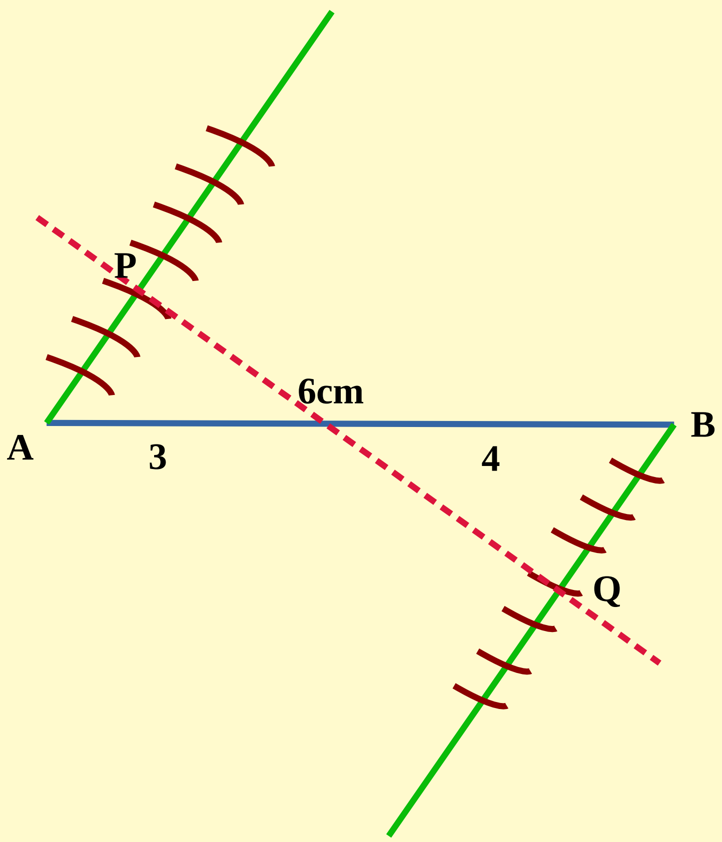
That is, their angles are equal

So the length of the sides changes at the same ratio

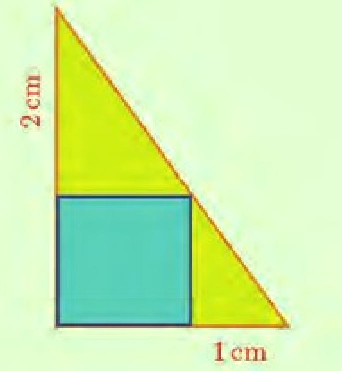
$\therefore \frac{AM}{BM} = \frac{PM}{QM}$

ii) Since the angles are equal in ΔAPM , ΔBQM $\frac{AM}{BM}=\frac{PM}{QM}=\frac{AP}{BQ}$.

iii) Draw a line 6 cm long. From A and B and upwards at the same angle
Draw downward angles. Divide these lines into 7 equal parts. Take 3 parts from A and 4 parts from B and mark P and Q and join them.



(4) The picture below shows a square sharing one corner with a right triangle and the other three corners on the sides of this triangle:

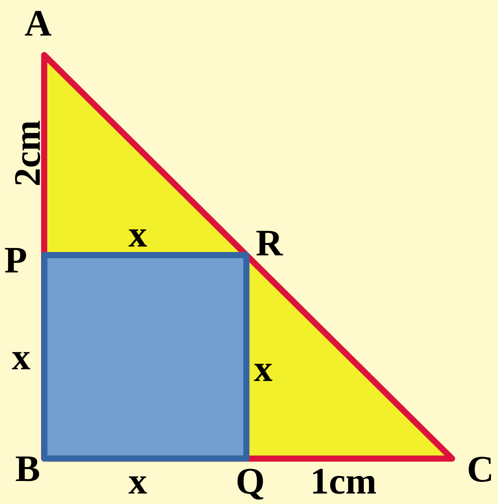


(i) Calculate the length of the sides of the square.

(ii) What is the length of the sides of such a square drawn within a triangle of sides 3 centimetres 4 centimetres, 5 centimetres?

Answer

i)



In the figure, PR and BC are parallel

In ΔAPR and ΔRQC

∠APR = ∠RQC

∠ARP = ∠RCQ (Since PR and BC are parallel)

The sides of ΔAPR and ΔRQC change at the same ratio

$$\frac{PR}{QC} = \frac{AP}{QR}$$

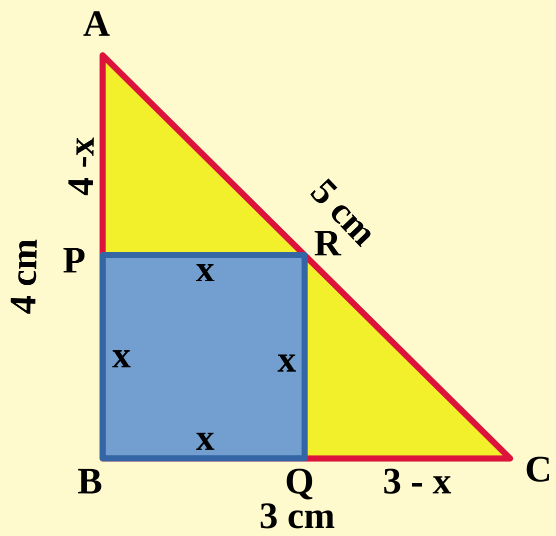
$$\frac{PR}{1} = \frac{2}{QR}$$

$$\frac{x}{1}=\frac{2}{x}$$

$x^2 = 2$

x= $\sqrt{2}$ cm.

ii)



Since the angles of right triangle APR and QCR are equal in the figure

$$\frac{x}{3-x}=\frac{4-x}{x}$$

$x^2 =(3-x)(4-x)$

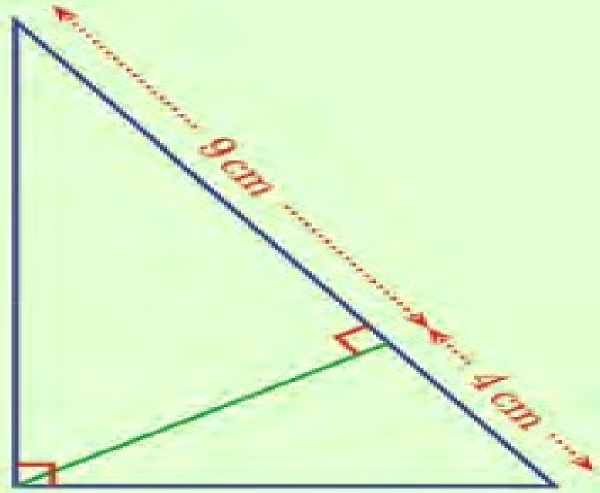
$x^2 = 12 - 3x - 4x + x^2$

$12 -7x = 0$

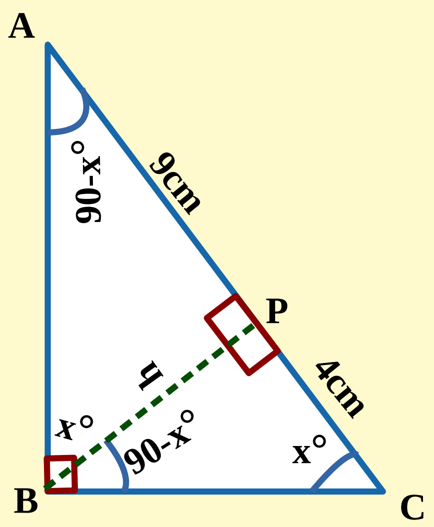
$7x = 12$

x= $\frac{12}{7}$ cm

(5) Calculate the area of the largest right triangle in the picture below:



Answer



The angles ΔABP and ΔBPC are equal

$$\frac{AP}{BP}=\frac{BP}{CP}$$

$$\frac{9}{h}=\frac{h}{4}$$

$h^2 = 36$

$h = \sqrt{36} = 6cm$

Area of large right triangle ABC= $\frac{1}{2} \times AC \times BP$

$= \frac{1}{2} \times 13 \times 6$

$= 39\ cm^2$

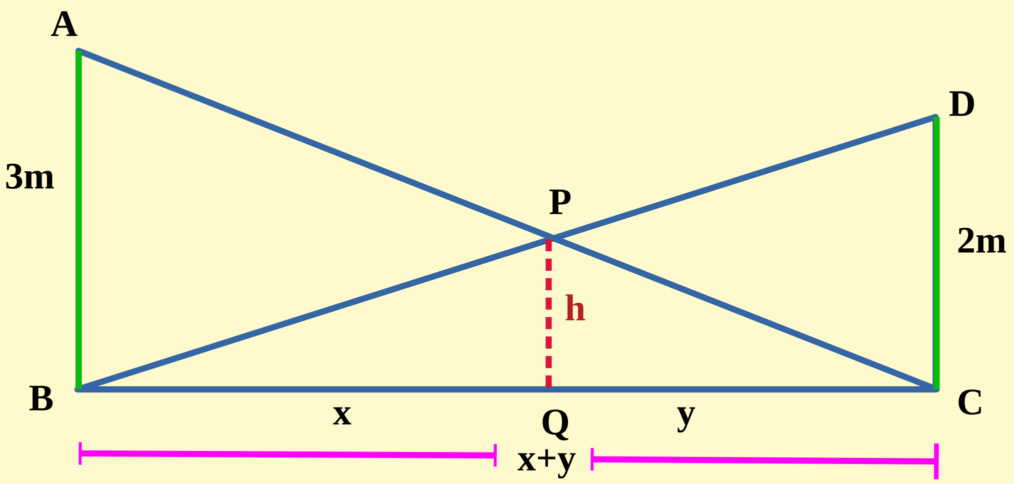
(6) Two poles of heights 3 metres and 2 metres are erected upright on the ground and ropes are stretched from the top of each to the foot of the other:

(i) At what height above the ground do the ropes cross each other ?

(ii) Prove that this height would be the same whatever the distance between the poles.

(iii) Denoting the heights of the poles as a, b and the height of the point of crossing above the ground as h, find the relation between a, b and h.

Answer



(i)

If ΔABC ,ΔPQC are considered

$\angle$B=$\angle$PQC=90°

$\angle$C=$\angle$C

The angles in ΔABC and ΔPQC are equal

$\therefore \quad \frac{h}{3}=\frac{y}{x+y}$ -------- (1)

Considering ΔBCD and ΔBQP, their angles are equal

$\therefore \quad \frac{h}{2}=\frac{x}{x+y}$ -------- (2)

Equation (1) +(2)

$$\frac{h}{3}+\frac{h}{2}=\frac{y}{x+y}+\frac{x}{x+y}$$

$$\frac{2h+3h}{6}=\frac{x+y}{x+y}$$

$$\frac{5h}{6}=1$$

5h =6

h = $\frac{6}{5}$ cm.

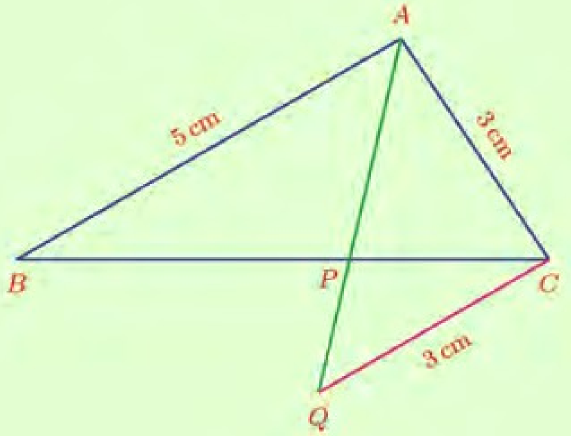
(ii) Whatever the value x +y , $\frac{h}{3}+\frac{h}{2}=1$. So the height does not change.

(iii) If the lengths of the poles are 3 m and 2 m, replace 'a' and 'b'

$$\frac{h}{a}+\frac{h}{b}=\frac{x+y}{x+y}=1$$

The relation h = $\frac{ab}{a+b}$ is obtained.

(7) In the picture below, AP is the bisector of $\angle A$ of triangle ABC:

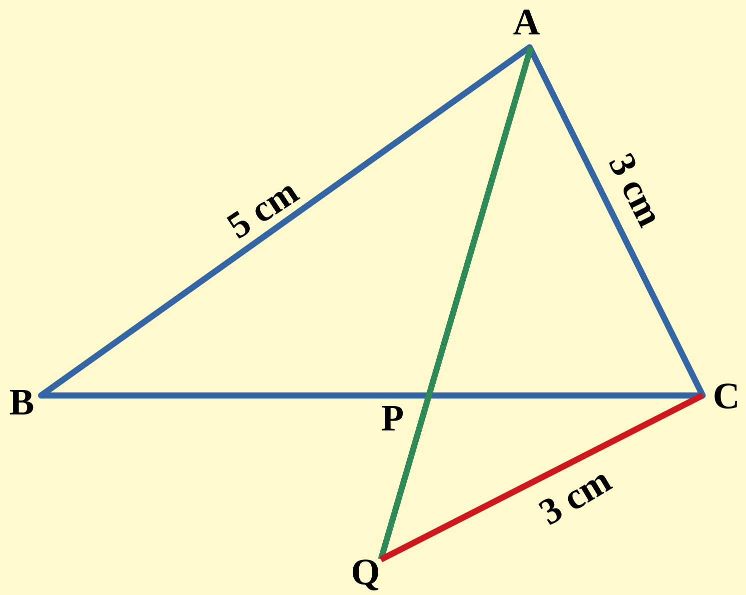


(i) Prove that angles of the triangles ABP and CPQ are the same.

(ii) Calculate $\frac{BP}{PC}$

(iii) Prove that in any triangle, the bisector of an angle divides the opposite side in the ratio of the sides containing the angle.

Answer



(i) Since AC = CQ =3cm in ΔACQ

$\angle$Q = $\angle$CAQ = x°

AP is the bisector of $\angle$A

$\angle$CAP = $\angle$BAP = $\angle$Q = x°

$\angle$APB = $\angle$CPQ (opposite angles)

That means two angles ΔABP and ΔCPQ are equal

So the angles are all equal.

(ii) Since the angles ΔABP and ΔCPQ are equal

$$\frac{BP}{PC} = \frac{AB}{CQ} = \frac{5}{3}$$

(III) $$\frac{BP}{PC} = \frac{AB}{CQ} = \frac{AB}{AC}$$

$$\frac{BP}{PC} = \frac{AB}{AC}$$

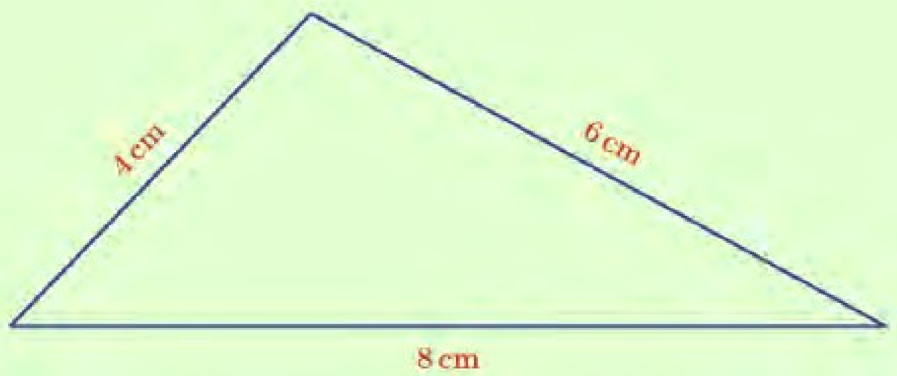
that is,

in any triangle, the bisector of an angle divides the opposite side in the ratio of the sides containing the angle.

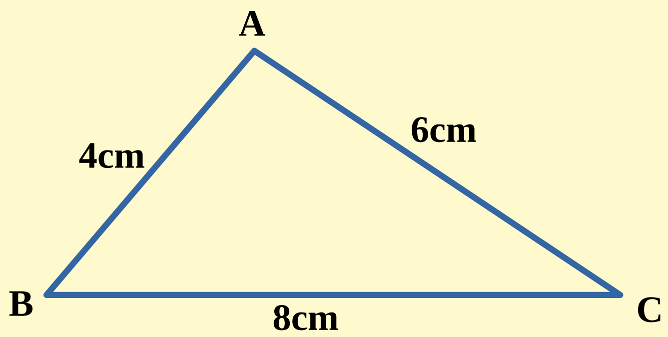
If two triangles have their sides scaled by the same factor, then their angles are the same.

Page no: 112

(1) Draw the triangle with angles the same as those of the triangle shown below, and sides scaled by $1\frac{1}{4}$.



Answer



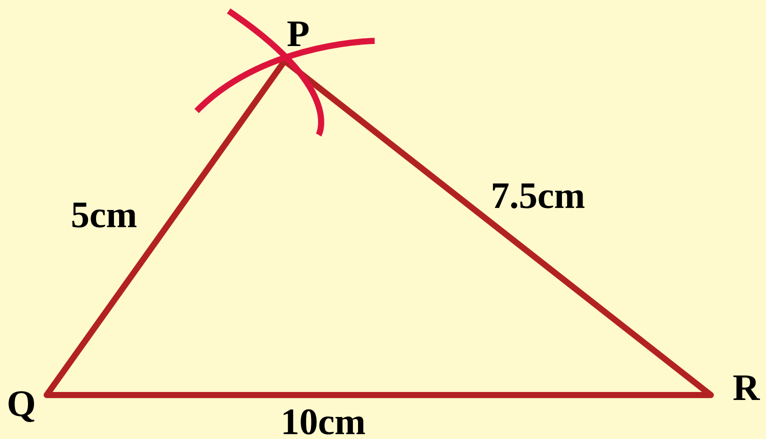
$1\frac{1}{4}$ times the sides AB=4cm , BC=8cm and AC=6cm are

PQ= $4\times\frac{5}{4}$ = 5cm

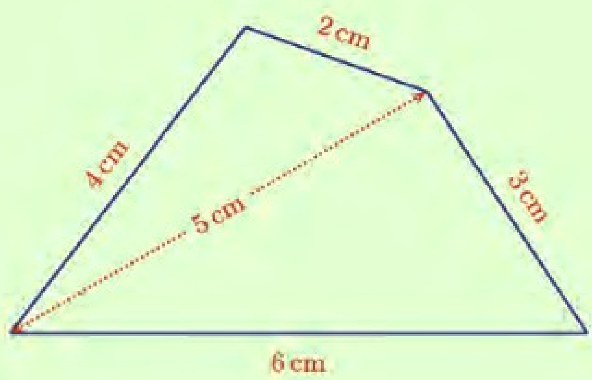
QR= $8\times\frac{5}{4}$ = 10cm

PR= $6\times\frac{5}{4}$ = 7.5cm

Draw ΔPQR whose sides are PQ=5cm, QR=10cm and PR=7.5cm



(2) See the picture of the quadrilateral.

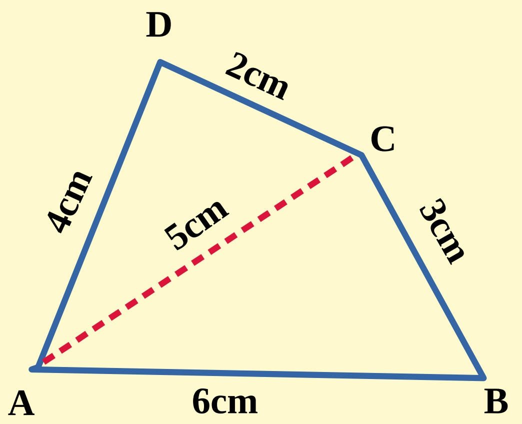


(i) Draw the quadrilateral with the same angles as this and sides scaled by a factor of $1\frac{1}{2}$.

(ii) Draw a quadrilateral with angles different from this and sides scaled by a factor of $1\frac{1}{2}$.

Answer

(i)



$1\frac{1}{2}$ times the sides of the sides are

PQ= $6 \times \frac{3}{2}$ = 9cm

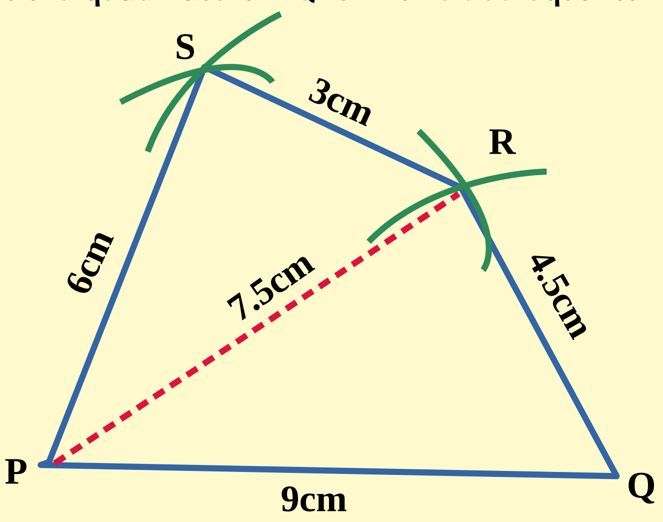
QR= $3 \times \frac{3}{2}$ = 4.5cm

RS= $2 \times \frac{3}{2}$ = 3cm

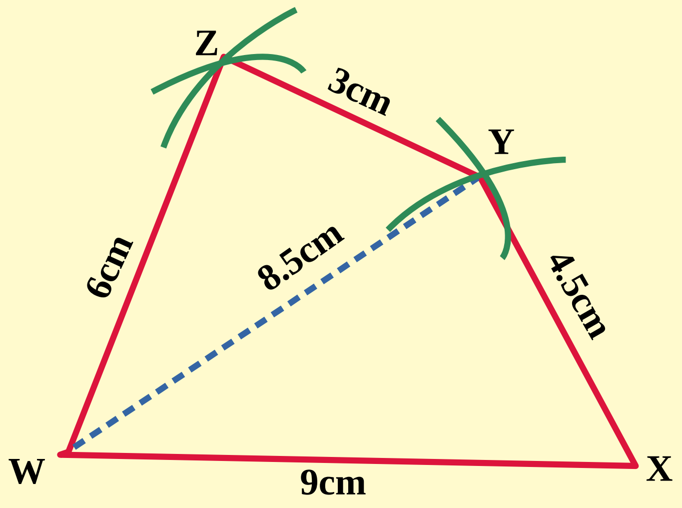
PS= $4 \times \frac{3}{2}$ = 6cm

PR = $5 \times \frac{3}{2}$ = 7.5cm

Construct the quadrilateral PQRS with sides equal to $1\frac{1}{2}$ of the sides



(ii) Draw all the sides 1.5 times(9cm,4.5cm, 3cm,6cm) but diagonal not measuring 7.5 cm.



(3) The area of a triangle is 6 square centimetres. What is the area of the triangle with lengths of sides four times those of this? What about the one with lengths of sides half of this?

Answer

(the scale factor of area is the square of the scale factor of the sides)

Area = $6cm^2$

If the sides are 4 times, area = $6 \times 4^2 = 6 \times 16 = 96cm^2$

If the sides are half of this, area = $6 \times \left(\frac{1}{2}\right)^2$

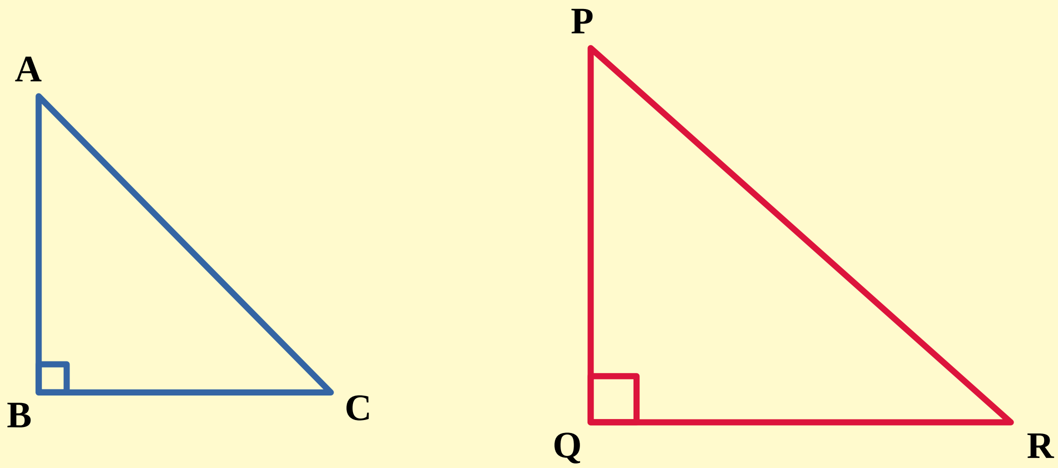
$= 6 \times \frac{1}{4} = \frac{6}{4} = 1.5\ cm^2$

If two triangles have two of the sides scaled by the same factor and the included angles equal, then the third sides are also scaled by the same factor and the other two angles are also equal.

Page no: 117 & 118

(1) Prove that if the perpendicular sides of two right triangles are scaled by the same factor, then the hypotenuses are also scaled by the same factor.

Answer



If we consider right triangle ABC and PQR

$\frac{AB}{PQ} = \frac{BC}{QR}$ (given)

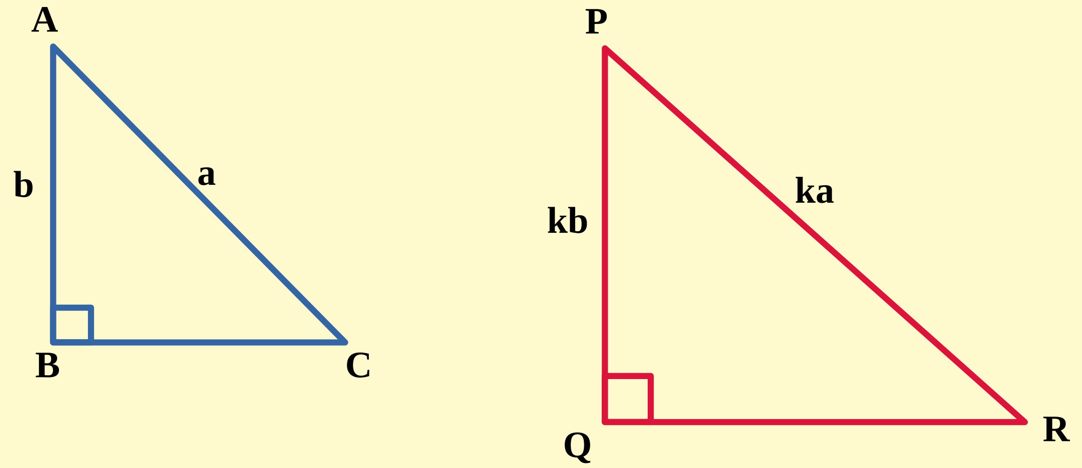
∠B=∠Q= 90°

The change in both sides is the same ratio and the angles between them are the same ,then the triangles are similar.

Therefore, the change in the third sides AC and PR is also the same ratio.

(2) Prove that if any two sides of two right triangles are scaled by the same factor, then the third side also is scaled by the same factor.

Answer



In ΔABC

BC= $\sqrt{AC^2 - AB^2}$

BC= $\sqrt{a^2 - b^2}$

In ΔPQR

QR= $\sqrt{PR^2 - PQ^2}$

QR= $\sqrt{(ka)^2 - (kb)^2}$

QR= $\sqrt{k^2 a^2 - k^2 b^2}$

QR= $\sqrt{k^2 (a^2 - b^2)}$

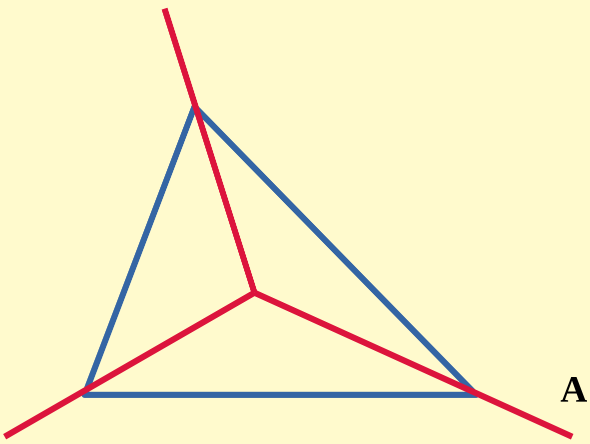
QR= k $\sqrt{a^2 - b^2}$

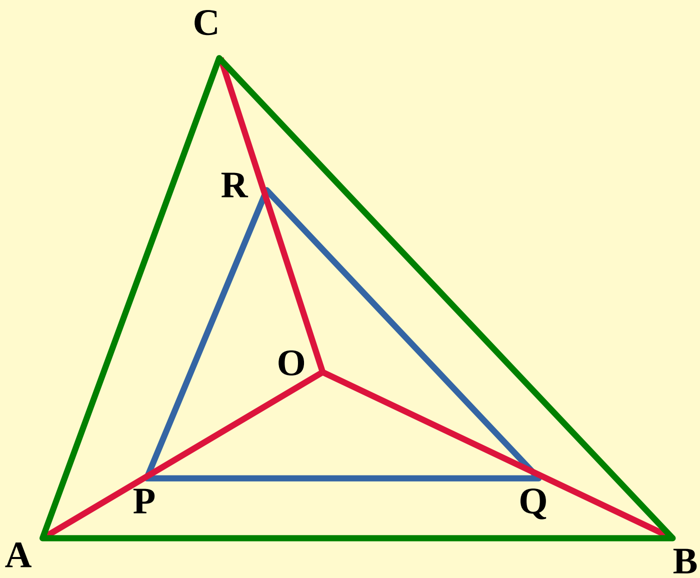
From this the change on the third side is also on the same scale.

(3) Draw a triangle and mark a point inside it. Join this point to the vertices and extend each of them by half its original length. Join the end points of these lines to form another triangle;

Prove that the sides of the larger triangle are one and a half times the sides of the original triangle.

Answer





In ΔOPQ , ΔOAB $OA = 1\frac{1}{2}\ OP$, $OB = 1\frac{1}{2}\ OQ$

That means the change on both sides is the same ratio.

And $\angle AOB = \angle POQ$

$\therefore AB = 1\frac{1}{2}\ PQ$.

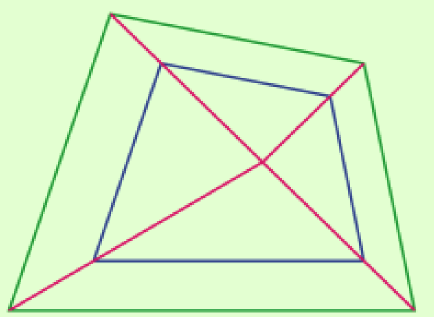
Like this we get,

BC= $1\frac{1}{2}$ QR

AC= $1\frac{1}{2}$ PR

the sides of the larger triangle are one and a half times the sides of the original triangle.

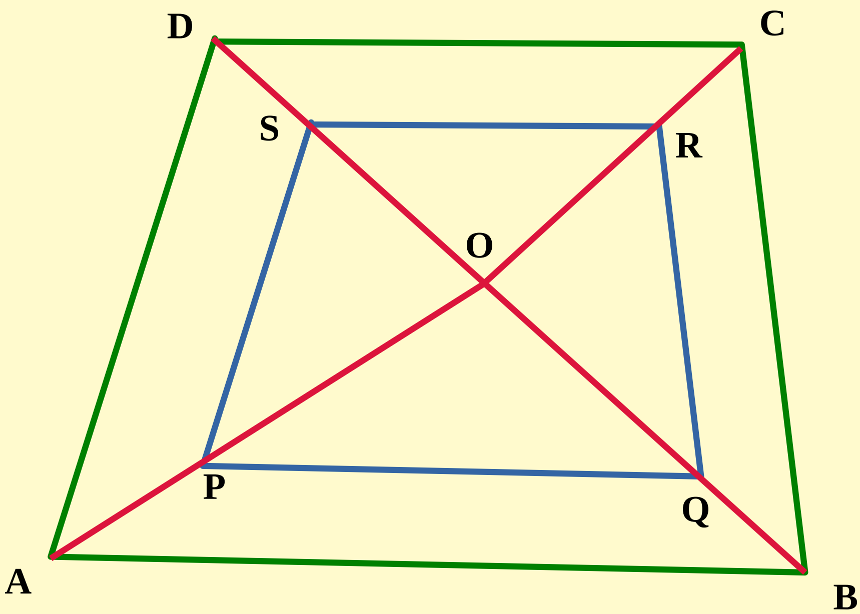
(4) The vertices of a quadrilateral are joined to a point inside it and these line are extended by the same scale factor. The ends of these lines are joined to form another quadrilateral:



(i) Prove that the sides of the two quadrilaterals also are scaled by the same factor.

(ii) Prove that the angles of the two quadrilaterals are the same.

Answer



In ΔOPQ , ΔOAB

OA = k×OP , OB = k × OQ

That means the change on both sides is the same ratio.

And ∠AOB = ∠POQ

That is, the change of two sides of these triangles is equal and the angles between them are equal.

∴ AB = k × PQ

and ∠OPQ = ∠OAB

∠OQP = ∠OBA

Like this we see that,

BC = k × QR

CD = k × SR

AD = k × PS

That is, the sides of the larger quadrilateral are equal to the sides of the smaller quadrilateral scaled up, and the angles of these four pairs of triangles is equal.

∴The angles that meet the corners of the quadrilateral PQRS are of the quadrilateral ABCD will be equal to the angles coming from the corners.