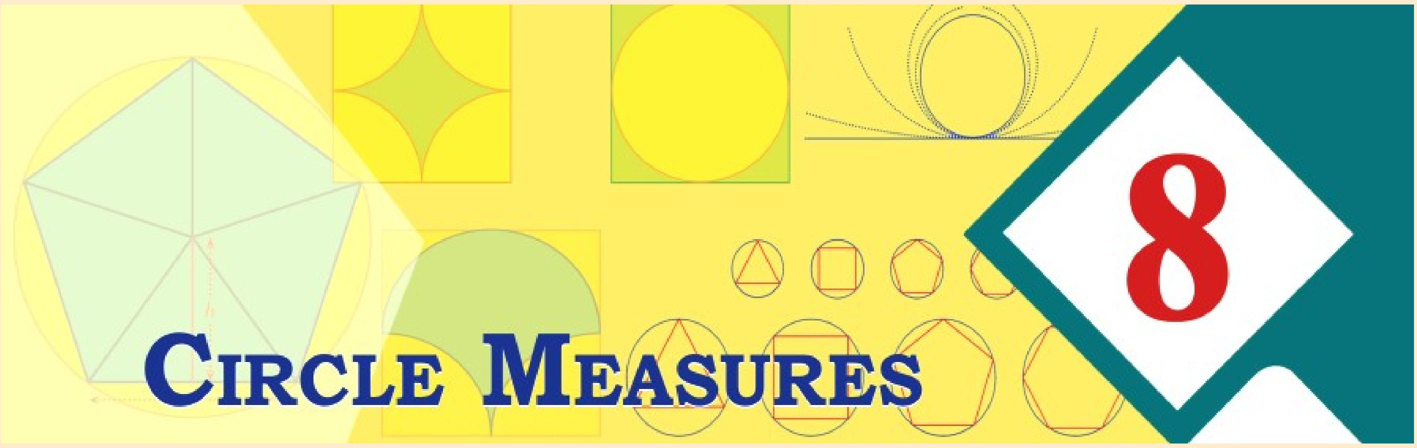

The circumference of circles are scaled by the same factor as their diameters

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(1) The circumference of a circle of diameter 2metres is measured and found to be 6.28 metres.

(i) How do we compute the circumference of a circle of diameter 4 metres, without actually measuring it?

(ii) What about the circumference of a circle of diameter 1metre?

(iii) And the circumference of a circle of diameter 3metre?

Answer

Circumference of a circle whose diameter is 2 m = 6.28m

(i) **The circumference of a circle whose diameter is 4 m= $2 \times 6.28 = 12.56$ m**

(2 times the diameter)

(ii) **The circumference of a circle whose diameter is 1 m= $\frac{1}{2} \times 6.28 = 3.14$m**

(Half diameter)

(iii) **The circumference of a circle whose diameter is 3m= $1\frac{1}{2} \times 6.28$**

$$= \frac{3}{2} \times 6.28 = 9.42\text{m}$$ **(1 $\frac{1}{2}$ times the diameter)**

(2) A piece of wire is bent into a circle of diameter 4 centimetres. If a wire of half the length is bent into a circle, what would be its diameter ?

Answer

When the circumference is halved, the diameter is also halved.

∴ Diameter = $\frac{1}{2} \times 4 = 2$cm

The circumference of a circle is π times its diameter.

The circumference of a circle is 2π times its radius.

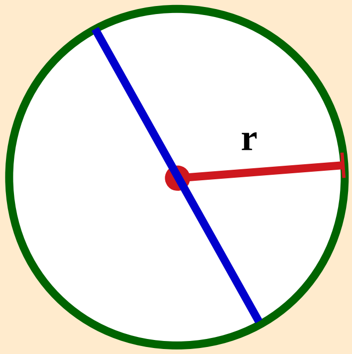
Circumference of the circle = $2\pi r$

OR

Circumference of the circle = πd

Diameter (d) = $2r$

Approximate value of $\pi \approx 3.14$



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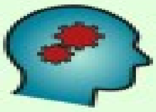
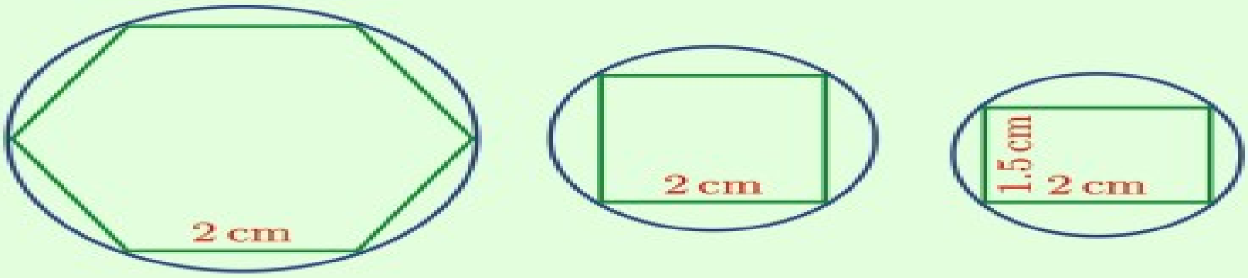
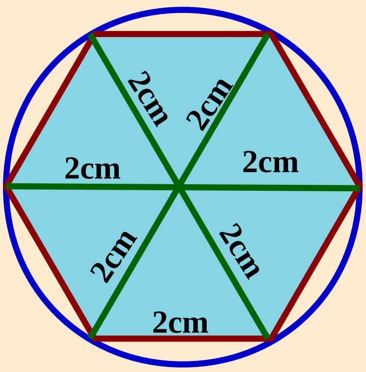


(1) Calculate the circumferences of the circles shown below:



Answer

(i)

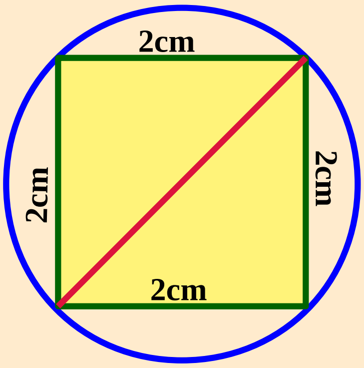


Side length of regular hexagon = radius of circle (r) = 2cm

Circumference of the circle= $2\pi r$

$\therefore$ Circumference of the circle= $2\pi \times 2 = 4\pi$ cm

(ii)



Diagonal of square = diameter of circle

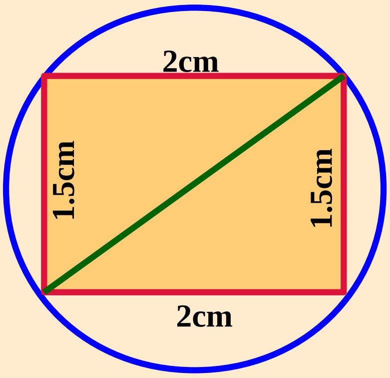
diameter (d) $= \sqrt{2^2+2^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Circumference of the circle= $\pi d = \pi \times 2\sqrt{2}$

$\therefore$ **Circumference of the circle=** $2\sqrt{2}\,\pi$ **cm**

(iii)



Diagonal of rectangle = diameter of circle

diameter (d) = $\sqrt{2^2+1.5^2}$

$= \sqrt{4+2.25} = \sqrt{6.25}$

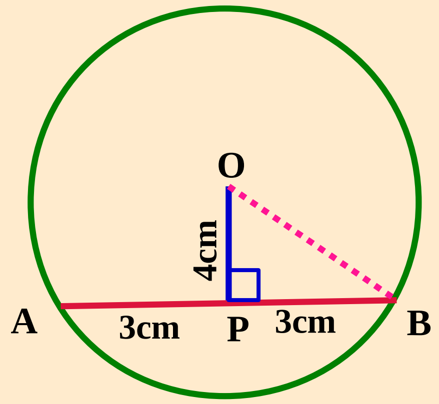
= 2.5cm

Circumference of the circle= $\pi d = \pi \times 2.5$

$\therefore$ **Circumference of the circle =** $2.5\,\pi$ **cm**

(2) In a circle, a chord 4 centimetres away from the centre is 6 centimetres long. What is the circumference of the circle?

Answer



OP = 4cm

PB= $\frac{1}{2} \times 6 = 3$**cm**

From the right triangle OPB

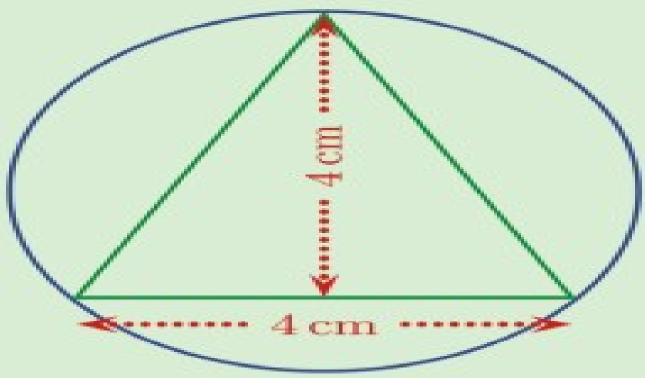
radius (r) = OB $= \sqrt{3^2+4^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$**cm**

Circumference of the circle= 2πr

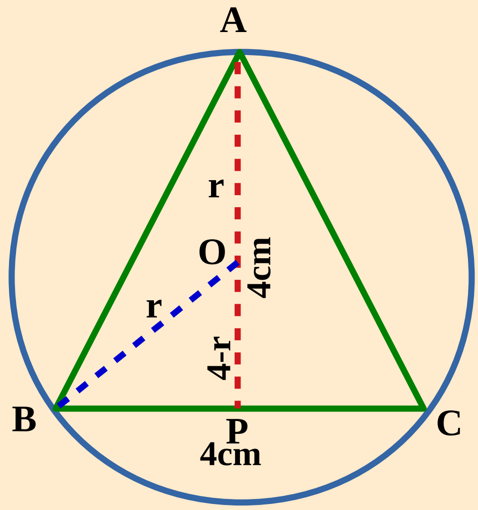
∴ Circumference of the circle= 2π × 5 = 10 π cm

(3) The figure below shows an isosceles triangle of base and height 4 centimetres drawn with vertices on a circle:



Calculate the circumference of the circle.

Answer



Since it is an isosceles triangle, the vertex will be the perpendicular bisector of the foot.

∴ BP= $\frac{4}{2}$ =2cm

If radius = r

In right triangle OPB , OP=4-r , OB=r

$r^2 = (4-r)^2 + 2^2$

$r^2 = 4^2-2×4×r + r^2+4$

$r^2 = 16-8r+ r^2 +4$

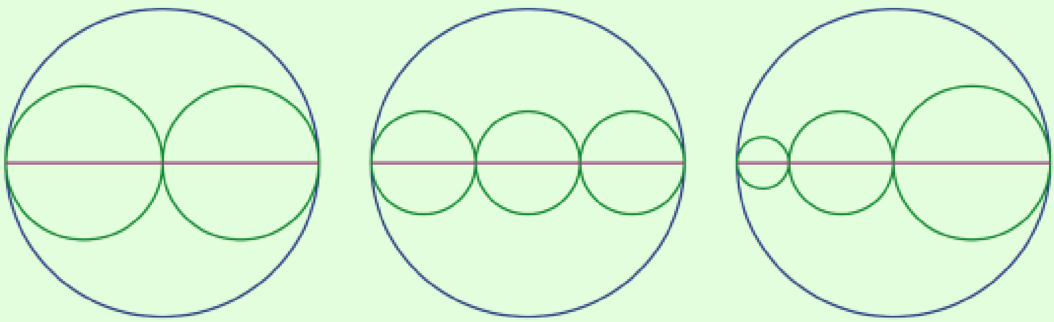
0 = 20-8r

8r=20

∴ r = $\frac{20}{8}$ = $\frac{5}{2}$

∴ Circumference of the circle = 2π × $\frac{5}{2}$ = 5π cm.

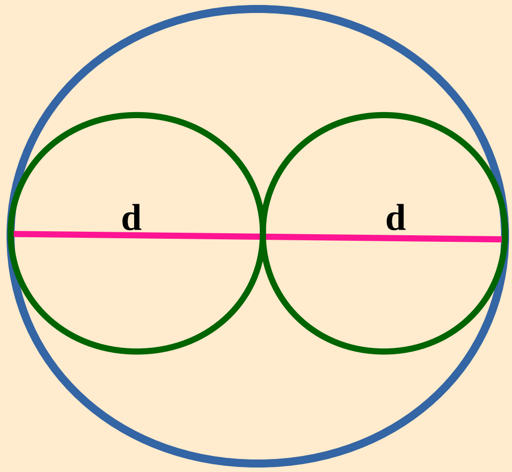
(4) In each of the pictures below, the centres of the large and small circles are on the same line. In the first and the second pictures, all the small circles have the same diameter:



In each of these figures, show that the circumference of the large circle is sum of the circumferences of the small circles.

Answer

(i)



If the diameter of the small circle is d

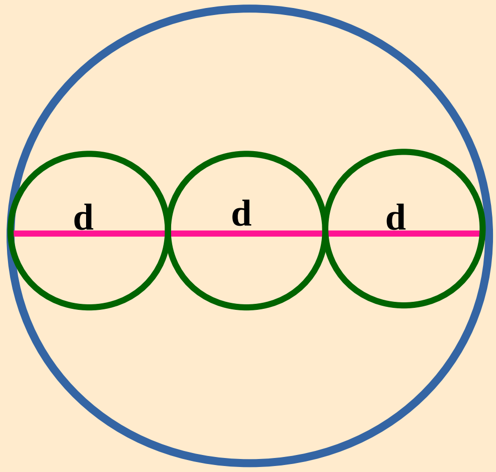
The circumference of a small circle= πd

∴ The perimeter of the two smaller circles= πd + πd = 2πd

Diameter of the large circle = 2d

∴ The circumference of the large circle= 2πd

(ii)



If the diameter of the small circle is d

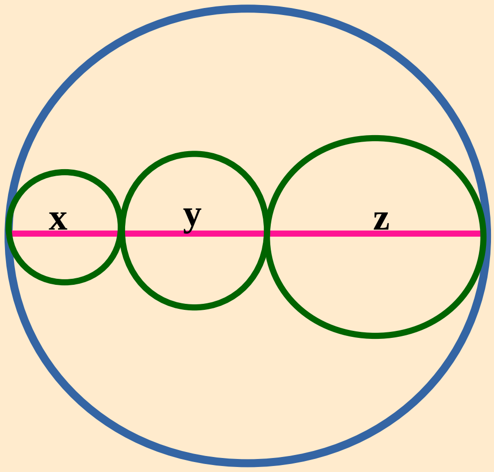
The circumference of a small circle= πd

∴ The perimeter of the three smaller circles= πd + πd +πd = 3πd

Diameter of the large circle= 3d

∴ The circumference of the large circle= 3πd

(iii)



If the diameters of the small circles are x, y, z in order of their size

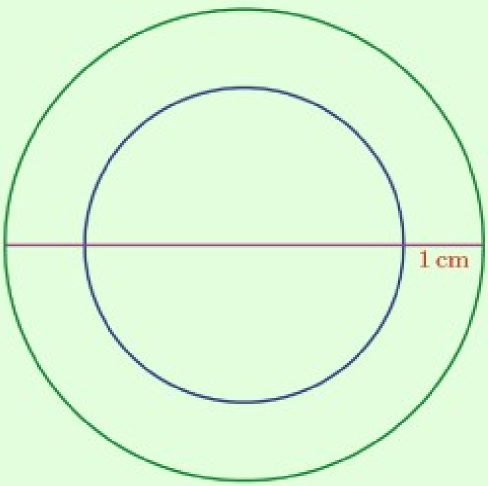
Sum of the circumferences of small circles = $\pi x + \pi y + \pi z = \pi(x+y+z)$

Diameter of the large circle= $x+y+z$

∴ The circumference of the large circle= $\pi(x+y+z)$

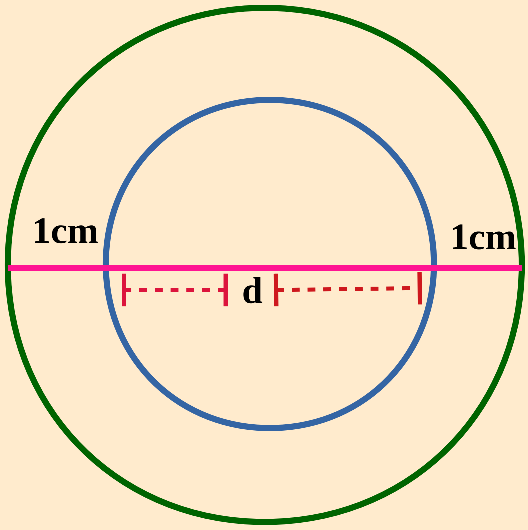
From the three figure, the circumference of the large circle is sum of the circumferences of the small circles.

(5) In the picture below, the two circles have the same centre.



How much more is the circumference of the larger circle than that of the smaller circle?

Answer



If the diameter of the small circle is d

The circumference of a small circle= πd

Diameter of the large circle= d+1+1 = d+2

The circumference of the large circle = π(d+2)

Difference of circumferences = π(d+2) - πd

= πd+2π-πd

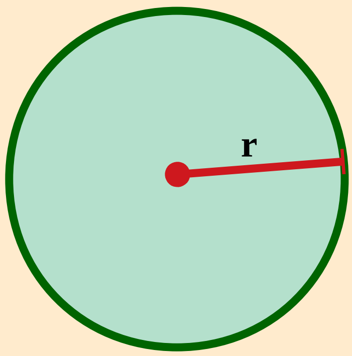
∴ Difference of circumferences = 2π cm

AREA OF CIRCLE

The area of a circle is half the product of its circumference and radius

The area of a circle is π times the square of the radius

Area of circle = πr^2



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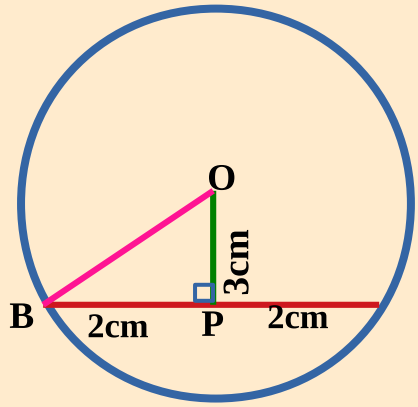
(1) The length of a chord of a circle, 3 centimetres from the centre, is 4 centimetres. What is the area of the circle?

Answer



In right triangle OPB

radius (r) OB = $\sqrt{3^2+2^2}$

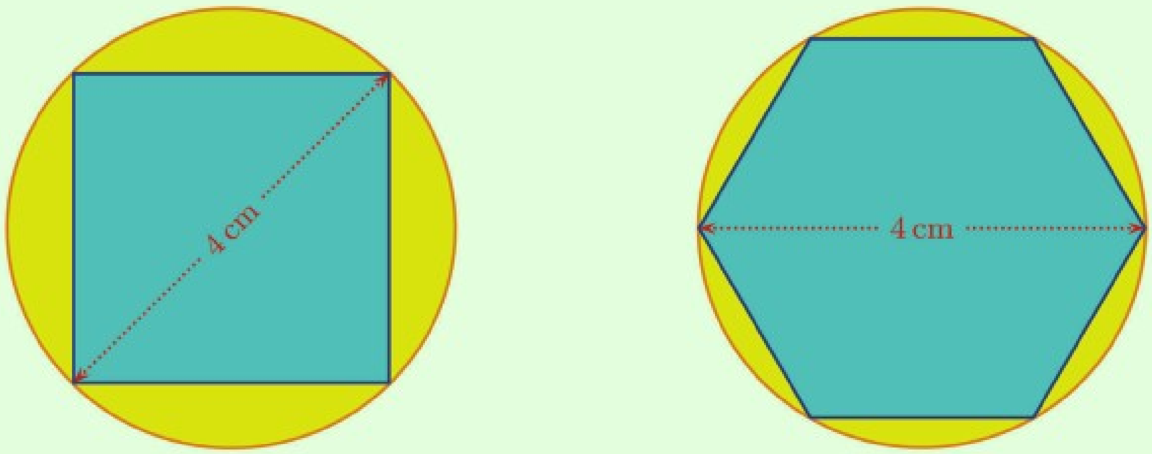
= $\sqrt{9+4} = \sqrt{13}$

Area of circle = πr^2

Area of circle = $\pi \left(\sqrt{13}\right)^2$

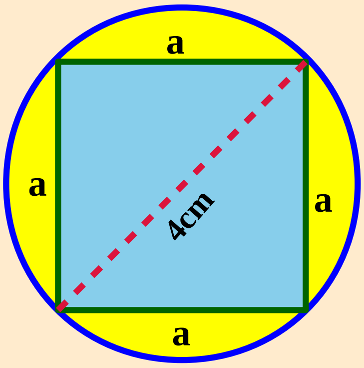
$\therefore$ Area of circle = 13π cm^2

(2) In each of the pictures below, compute the difference between the area of the circle and the area of the regular polygon, correct up to two decimal places:



Answer

(i)



If the sides of the square are 'a'

$a^2 + a^2 = 4^2$

$2a^2 = 16$

$a^2 = \frac{16}{2} = 8$

side 'a' = $\sqrt{8}$

Area of the square= $a^2 = (\sqrt{8})^2 = 8cm^2$

Diameter of the circle = 4cm

radius (r) = 2cm

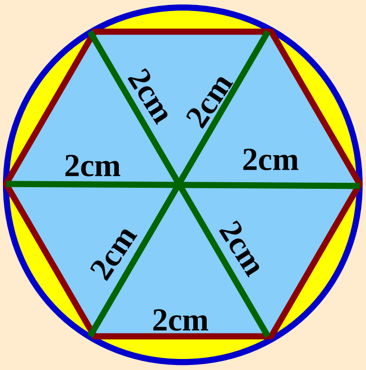
Area of circle= πr^2

Area of circle= $\pi \times 2^2 = 4\pi$

$= 4 \times 3.14 = 12.56\ cm^2$

$\therefore$ Difference between the area = $12.56 - 8 = 4.56\ cm^2$

(ii)



Area of circle = $\pi \times 2^2 = 4\pi$

$= 4 \times 3.14 = 12.56\ cm^2$

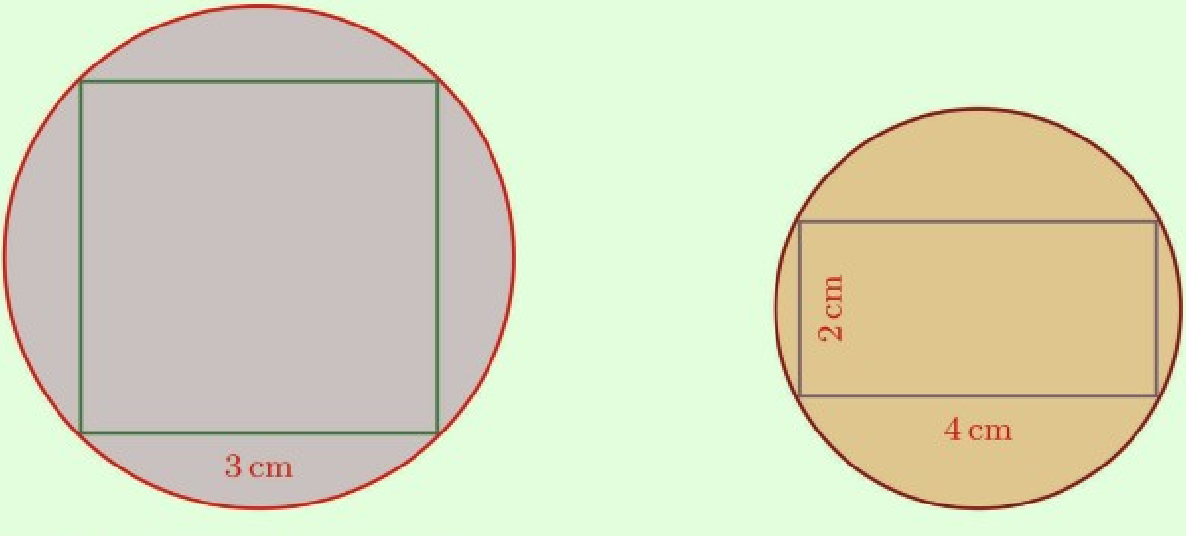
Side of regular hexagon= 2cm

Area of regular hexagon= $\frac{6 \times \sqrt{3} \times 2^2}{4} = 6\sqrt{3}\ cm^2$

Area of regular hexagon= $6 \times 1.73 = 10.38 cm^2$

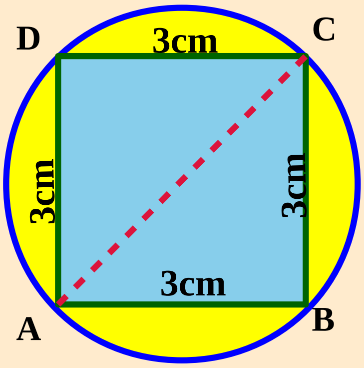
$\therefore$ Difference between the area = $12.56 - 10.38 = 2.18 cm^2$

(3) In the pictures below, circles are drawn through the vertices of a square and a rectangle:



Calculate the areas of the circles

Answer



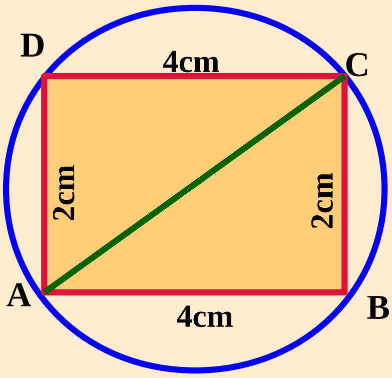
Diameter of the circle AC = $\sqrt{3^2+3^2} = \sqrt{18}$

Radius (r) = $\frac{\sqrt{18}}{2}$

Area of circle = $\pi \times \left(\frac{\sqrt{18}}{2}\right)^2$

∴ Area of circle = $\frac{9}{2}\pi$ cm²

(II)



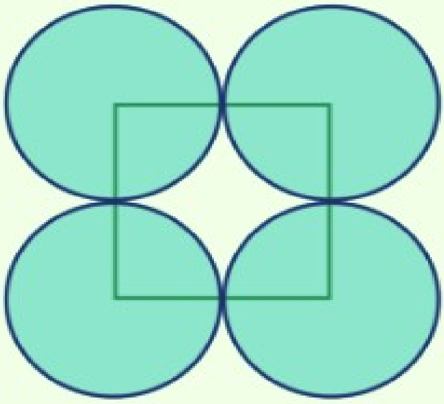
Diameter of the circle, AC = $\sqrt{4^2+2^2} = \sqrt{20} = 2\sqrt{5}$ cm

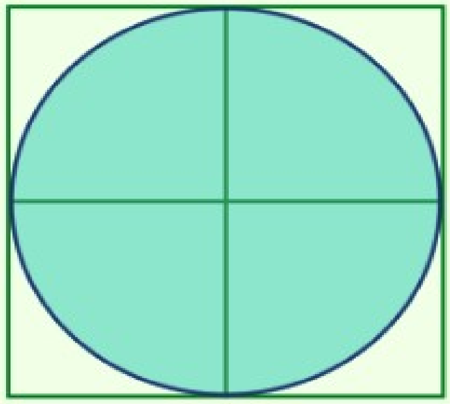
Radius (r) = $\frac{2\sqrt{5}}{2} = \sqrt{5}$

Area of circle= $\pi \times (\sqrt{5})^2$

∴ Area of circle = $5\pi cm^2$

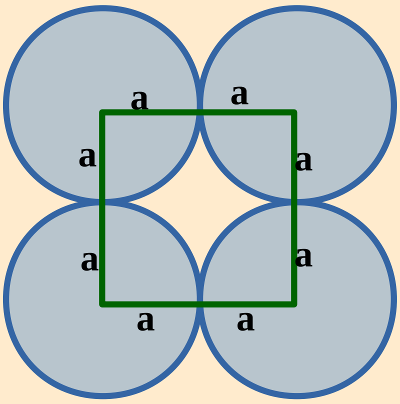
(4) Draw a square and draw circles with its vertices as centres and radius as half the side. Draw another square composed of four smaller squares of the same size as the first square, and draw a circle that just fits inside it.

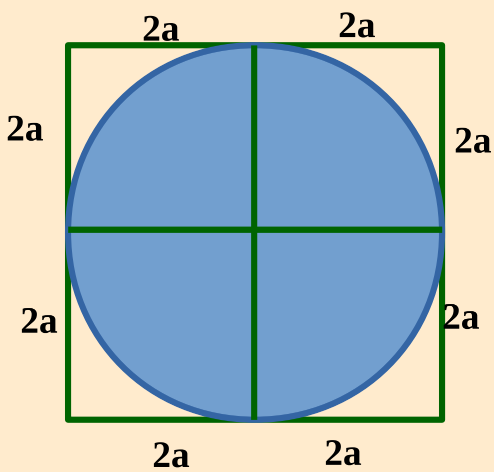




Prove that the area of the large circle is the sum of the areas of the four small circles.

Answer





In the first figure, if one side of the square is 2a, then the smaller circles , the radius will be ‘a’

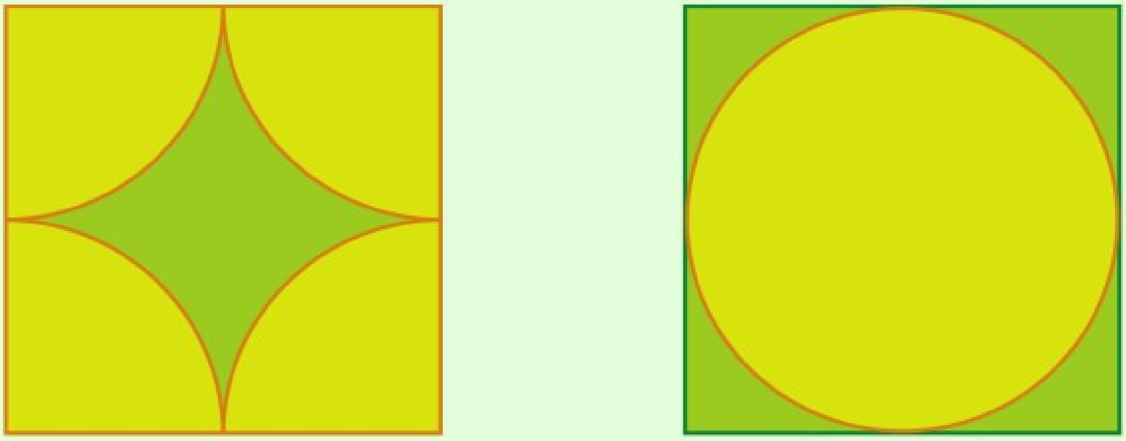
Area of 4 small circle = $4\times \pi\times a^2 = 4\pi a^2$

In the second figure, if one side of the square is 4a, then the circle’s radius will be ‘2a’

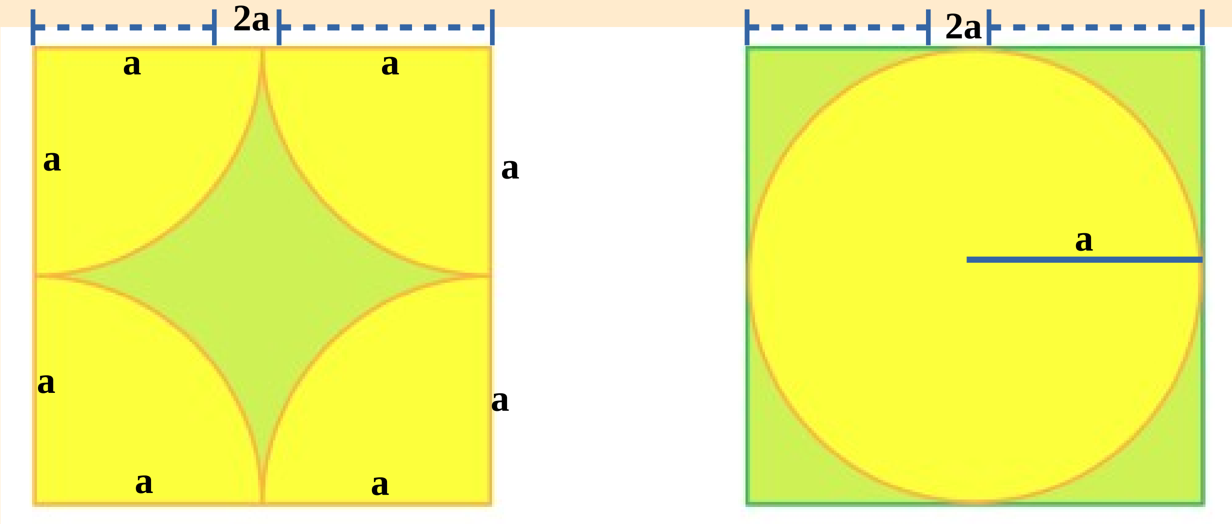
Area of large circle = $\pi\times(2a)^2 = 4\pi a^2$

∴ The area of the large circle is the sum of the areas of the four small circles.

(5) In the pictures below, the squares are of the same size. Prove that the areas of the green regions in the pictures are equal:



Answer



In the first figure, if one side of the square is 2a, then the radius is a

Area of green area = area of square - area of a circle

$= (2a)^2 - \pi a^2$

$= 4a^2 - \pi a^2$

$= a^2(4 - \pi)$ **Sq.unit----(1)**

Since one side of the square in the second figure is 2a, the radius will be 'a'

Area of green area = area of square - area of large circle

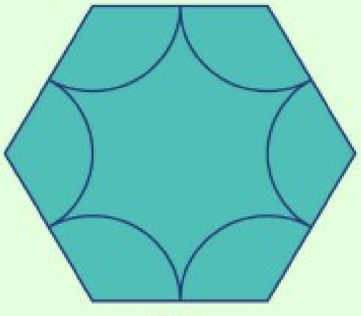
$= (2a)^2 - \pi a^2$

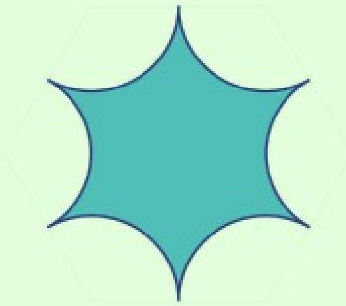
$= 4a^2 - \pi a^2$

$= a^2(4 - \pi)$ **Sq.unit----(2)**

∴ from (1) & (2) The green regions have the same area.

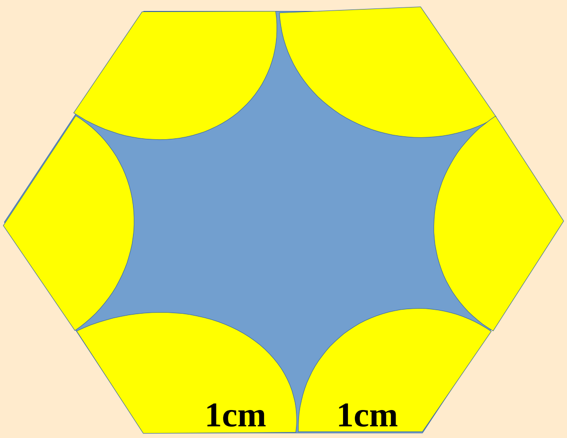
(6) Parts of circles are drawn with the vertices of a regular hexagon as centres and the figure below is cut out:





Calculate the area of the figure cut out.

Answer



If we join 6 circle's parts in the first picture, we will get 2 circles.

The radius of the circle = 1 cm

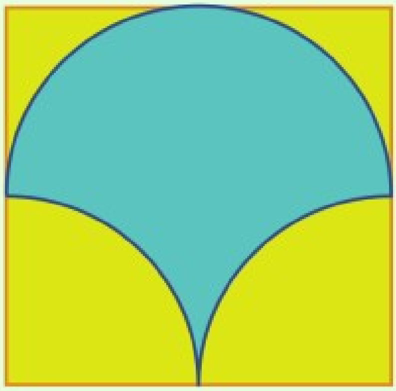
The area of two circles = $2\times \pi\times 1^2 = 2\pi cm^2$

Area of regular hexagon = $\frac{6\times\sqrt{3}\times 2^2}{4} = 6\sqrt{3}\ cm^2$

The area of the figure cut out = Area of regular hexagon - Area of 2 circles

∴The area of the figure cut out = $(6\sqrt{3} - 2\pi)\ cm^2$

(7) Parts of a circle are drawn within a square like this:



Prove that the area of blue region is half the area of the square.

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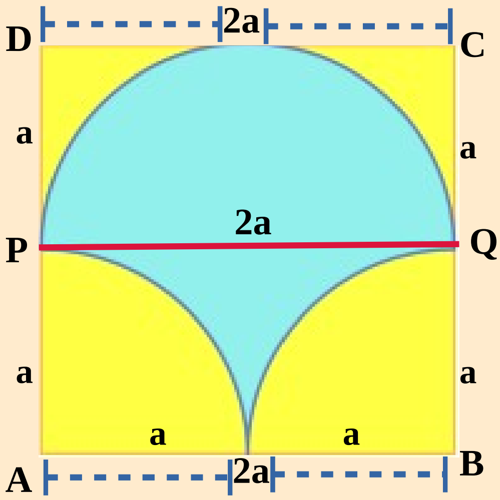

If one side of a square is 2a, then

Area of the square= $(2a)^2 = 4a^2$

Between the midpoints of the sides of the square as seen in the figure when matched, it becomes two rectangles.

Area of blue region in rectangle PQCD= $\frac{\pi \times a^2}{2}$ ------ (1)

Area of blue region in rectangle PQAB = Area of the rectangle – area of 2 circle part

$$= 2a^2 - \frac{\pi \times a^2}{2}$$

$$= \frac{4a^2 - \pi a^2}{2} \text{ ------ (2)}$$

The area of blue region = (1) + (2)

$$= \frac{\pi a^2}{2} + \frac{4a^2 - \pi a^2}{2}$$

$$= \frac{4a^2}{2} = 2a^2$$

∴ The area of blue region is half the area of the square.