9 REAL NUMBERS

If $x > 0$, then $|x| = x$

If $x < 0$, then $|x| = -x$

If $x = 0$, then $|x| = 0$

On the number line, the distance between the point labelled zero and the point labelled by another number is the absolute value of that number.

On the number line, the distance between the point labelled 0 and the point labelled x is $|x|$.

Any number and its negative have the same absolute value

$|-x| = |x|$ for any number x

$|x|^2 = x^2$ for any number x.

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(1) Complete the table below:

x	y	xy	$\|x\|$	$\|y\|$	$\|xy\|$	$\|x\|\,\|y\|$
4	3	12	4	3	12	12
−4	3					
4	−3					
−4	−3					

(i) Expand the table by taking some more pairs x, y of numbers. Do you see any relation between $|xy|$ and $|x|\,|y|$?

(ii) Prove that $|xy| = |x||y|$ for any two numbers x and y

Answer

x	y	xy	$\|x\|$	$\|y\|$	$\|xy\|$	$\|x\|$ $\|y\|$
4	3	12	4	3	12	12
-4	3	-12	4	3	12	12
4	-3	-12	4	3	12	12
-4	-3	12	4	3	12	12

Another pair

x	y	xy	$\|x\|$	$\|y\|$	$\|xy\|$	$\|x\|$ $\|y\|$
6	5	30	6	5	30	30
-6	5	-30	6	5	30	30
6	-5	-30	6	5	30	30
-6	-5	30	6	5	30	30

(i) From the table above we get $|xy| = |x|\ |y|$

(ii) To prove $|xy| = |x|\ |y|$

(a) If both are positive numbers, then x × y is positive
|xy|=xy and | x | = x, | y | =y
|xy|= xy = x × y = | x |×| y |

(b) If both are negative, then x × y is positive
|xy|=xy and | x | = -x, | y | = -y
|xy|= xy = -x × -y = | x |×| y |

(c) If x is positive and y is negative, then x × y is negative.
|xy|= -xy and | x | = x, | y | = -y
|xy|= -xy = x × -y = | x |×| y |

(d) If x is negative and y is positive, x × y is a negative number.
|xy|= -xy and | x | = -x, | y | = y
|xy|= -xy = -x × y = | x | × | y |

(2) Complete the table below:

x	y	$x+y$	$\|x\|$	$\|y\|$	$\|x\|+\|y\|$	$\|x+y\|$
3	5	8	3	5	8	8
−3	5	2	3	5	8	2
3	−5					
−3	−5					

Expand the table by taking some more pairs x, y of numbers. Do you see any relation between $|x|+|y|$ and $|x+y|$?

Answer

x	y	x+y	$\|x\|$	$\|y\|$	$\|x\|$ + $\|y\|$	$\|x+y\|$
3	5	8	3	5	8	8
-3	5	2	3	5	8	2
3	-5	-2	3	5	8	2
-3	-5	-8	3	5	8	8

Another pair

x	y	x+y	$\|x\|$	$\|y\|$	$\|x\|$ + $\|y\|$	$\|x+y\|$
4	6	10	4	6	10	10
-4	6	2	4	6	10	2
4	-6	-2	4	6	10	2
-4	-6	-10	4	6	10	10

From the above table we got the relationship $|x| + |y| \geq |x + y|$.

Distances

The distance between two numbers x and y on the number line is $|x-y|$

The distance between two numbers on the number line is the number got on subtracting the smaller number from the larger number.

$|x-y|$ is the algebraic form of subtracting the smaller of the numbers x, y from the larger.

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(1) Find x satisfying each of the equations below:

(i) $|x| = 5$ (ii) $|x-3| = 2$

(iii) $|x-2| = 3$ (iv) $|x+2| = 3$

Answer

(i). $|x| = 5$

$x = \pm 5$

$x = +5$ or -5

(ii). $|x-3| = 2$

$x-3 = \pm 2$

$x-3 = 2$

$x = 2+3 = 5$

$x-3 = -2$

$x = -2+3 = 1$

$\therefore x = 5$ or 1

(iii). $|x-2| = 3$

$x-2 = \pm 3$

$x-2 = 3$

$x = 3+2 = 5$

$x-2 = -3$

$x = -3+2 = -1$

$\therefore x = 5$ or -1

(iv). $|x+2| = 3$

$x+2 = \pm 3$

$x+2 = 3$

$x = 3-2 = 1$

$x+2 = -3$

$x = -3-2 = -5$

$\therefore x = 1$ or -5

(2) Find between which numbers x should lie to satisfy each of the equations below:

(i) $|x| < 3$ (ii) $|x-2| < 1$

(iii) $|x-1| < 2$ (iv) $|x+1| < 2$

Answer

(i) **| x | <3**
x < ±3
x lie in between -3 and +3

(ii) **| x-2 | < 1**
x < ±1+2
x < 1+2
x < 3
x < -1+2
x < 1
x lie in between 1 and +3

(iii) **| x−1 |<2**
x – 1 < ±2
x < ±2 +1
x < 2+1
x < 3
x < -2+1
x < - 1
x lie in between -1 and +3

(iv) **|x +1 | <2**
x +1 < ±2
x < ±2 -1
x < 2-1
x < 1
x < -2-1
x < - 3
x lie in between 1 and -3

(3) Find the integers satisfying each of the equations in problem (2)

Answer

(i) **-2,-1,0,1,2** **(ii)** **2**

(iii) **0,1,2** **(iv)** **-2,- 1,0**

Midpoint

The midpoint of two points on the number line is the point marked by half the sum of the numbers which mark those points

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(1) Find the number which mark the midpoint of the points marked by each pair of numbers given below on the number line:

(i) 1, −5 (ii) −1, −5 (iii) $-\frac{1}{2}, -\frac{1}{3}$

(iv) $-\frac{1}{2}, \frac{3}{4}$ (v) −2.5, 3.5 (vi) 1.3, 8.7

(vii) $-\sqrt{2}, -\sqrt{3}$ (viii) $-\sqrt{3}, \sqrt{7}$

Answer

(i) 1,-5

Midpoint= $\frac{1+(-5)}{2}=\frac{-4}{2}=-2$

(ii) -1 ,-5

Midpoint= $\frac{(-1)+(-5)}{2}=\frac{-6}{2}=-3$

(iii) $\frac{-1}{2}, \frac{-1}{3}$

Midpoint $= \frac{\frac{-1}{2}+\frac{-1}{3}}{2}$

$= \frac{\frac{-3+-2}{6}}{2}$

$= \frac{\frac{-5}{6}}{2}$

$= \frac{-5}{6\times 2}=\frac{-5}{12}$

(iv) $\frac{-1}{2}, \frac{3}{4}$

Midpoint $= \frac{\frac{-1}{2}+\frac{3}{4}}{2}$

$= \frac{\frac{1}{4}}{2}$

$= \frac{1}{4\times 2} = \frac{1}{8}$

(v) -2.5,3.5

Midpoint $= \frac{-2.5+3.5}{2} = \frac{1}{2} = 0.5$

(vi) 1.3, 8.7

Midpoint $= \frac{1.3+8.7}{2} = \frac{10}{2} = 5$

(vii) $-\sqrt{2}, -\sqrt{3}$

Midpoint $= \frac{-\sqrt{2}+-\sqrt{3}}{2} = \frac{-(\sqrt{2}+\sqrt{3})}{2}$

(viii) $-\sqrt{3}, \sqrt{7}$

Midpoint $= \frac{-\sqrt{3}+\sqrt{7}}{2} = \frac{(\sqrt{7}-\sqrt{3})}{2}$

(2) Find the numbers which mark the points dividing the distance between the points marked by 1 and 2 into four equal parts, on the number line.

Answer

The distance between 1 and 2 is 2 – 1 = 1

This distance can be divided into four equal parts.

The distance between each part $= \frac{1}{4} = 0.25$

Points $= 1 + 0.25 = 1.25 \left(1\ \frac{1}{4}\right)$

$= 1 + 0.50 = 1.50 \left(1\ \frac{1}{2}\right)$

$= 1 + 0.75 = 1.75 \left(1\ \frac{3}{4}\right)$

Find x satisfying each of the equations below:

(i) $|x-1| = |x-3|$ (ii) $|x-3| = |x-4|$

(iii) $|x+2| = |x-5|$ (iv) $|x| = |x+1|$

Answer

(i) $|x-1| = |x-3|$

$\therefore$ **The distance between x and 1, 3 is equal.**

$\therefore$ **x is between '1' and '3' i.e. x is the midpoint of 1 and 3**

$x = \frac{1+3}{2} = \frac{4}{2} = 2$

(ii) $|x-3| = |x-4|$

$\therefore$ **The distance between x and 3, 4 is equal.**

$\therefore$ **x is between '3' and '4' i.e. x is the midpoint of 3 and 4**

$x = \frac{3+4}{2} = \frac{7}{2} = 3.5$

(iii) $|x+2| = |x-5|$

$\therefore$ **The distance between x and -2, 5 is equal.**

$\therefore$ **x is between '-2' and '5' i.e. x is the midpoint of -2 and 5**

$x = \frac{-2+5}{2} = \frac{3}{2} = 1.5$

(iv) $|x| = |x+1|$

$\therefore$ **he distance between x and -1, 0 is equal.**

$\therefore$ **x is between '-1 ' and '0 ' i.e. x is the midpoint of -1 and 0**

$x = \frac{-1+0}{2} = \frac{-1}{2} = -0.5$