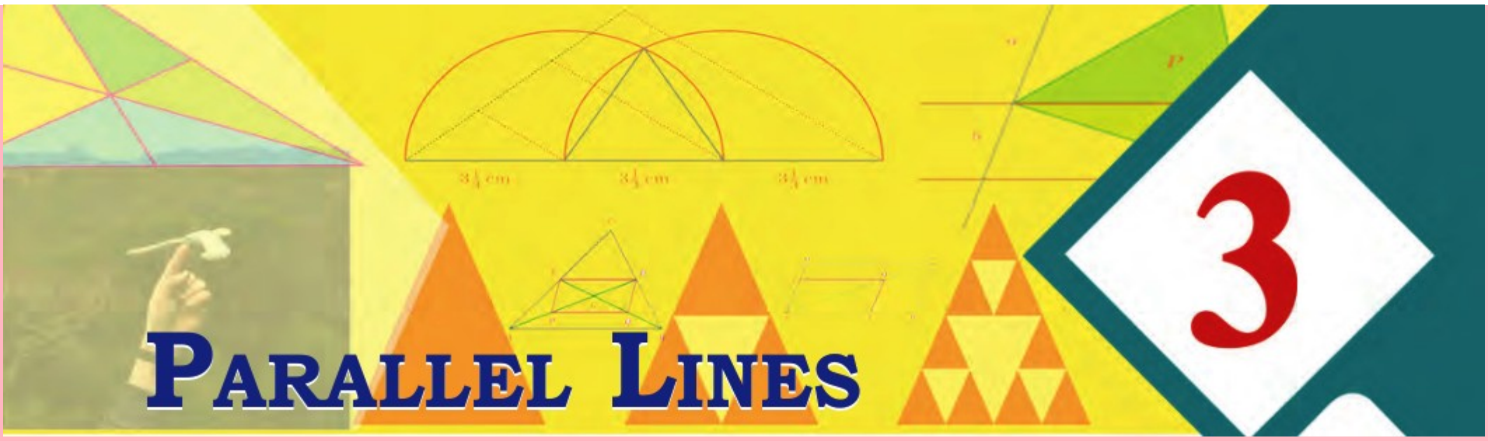
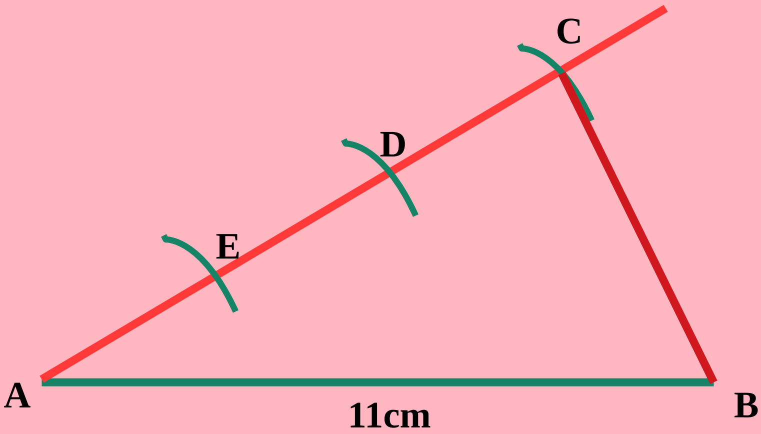


Page no: 39

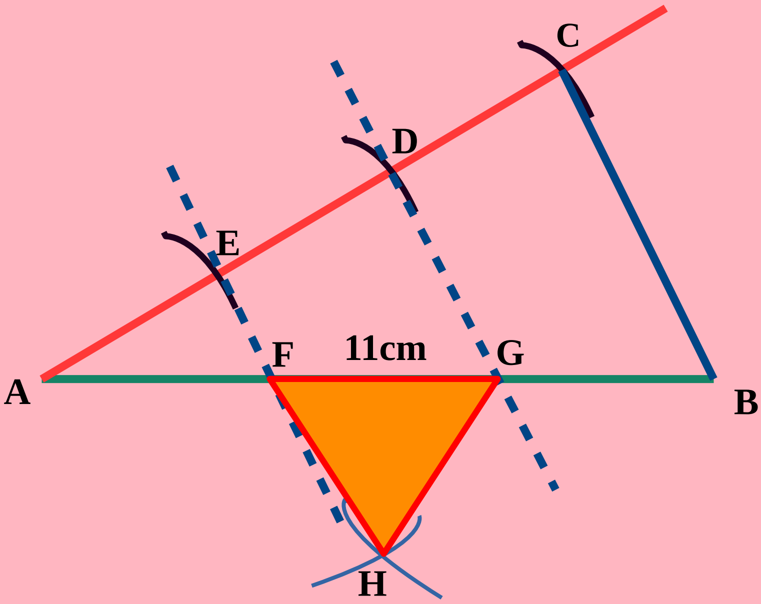
(1) *Draw an equilateral triangle of perimeter 11 centimetres.*

ANSWER

Draw a line AB of length 11cm. 3 equal parts from A to C by compass Join C and B and marking the points E and D.



Draw lines parallel to BC from E and D to AB. Then 3 equal parts in AB, draw an equilateral triangle FGH.

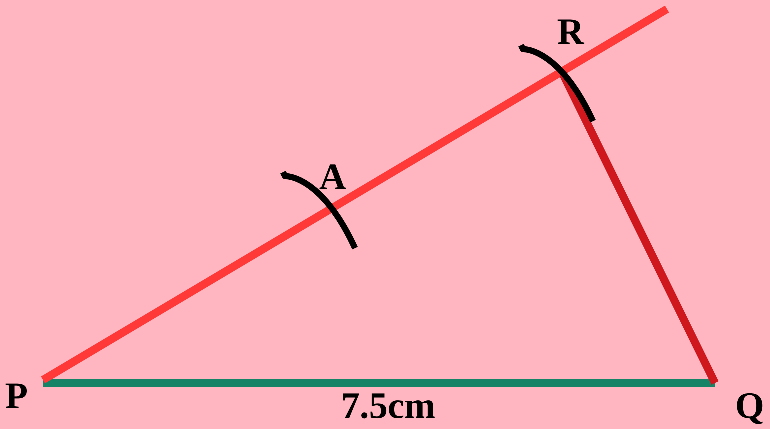


FH+FG+GH=11cm

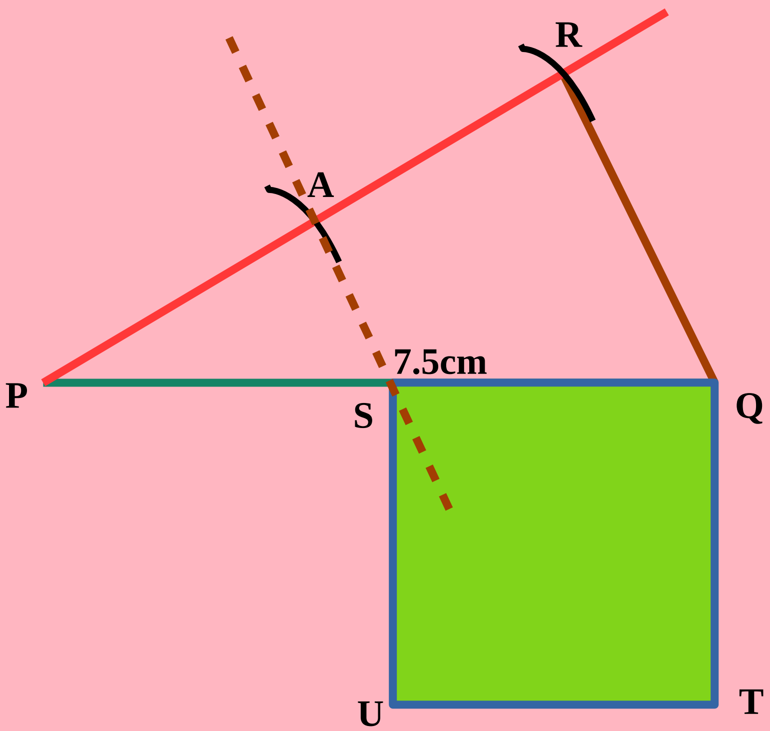
(2) ***Draw a square of perimeter 15 centimetres.***

ANSWER

Draw a line PQ of length 7.5cm. From P to R mark 2 equal parts by compass. Join R and Q and marking point A.



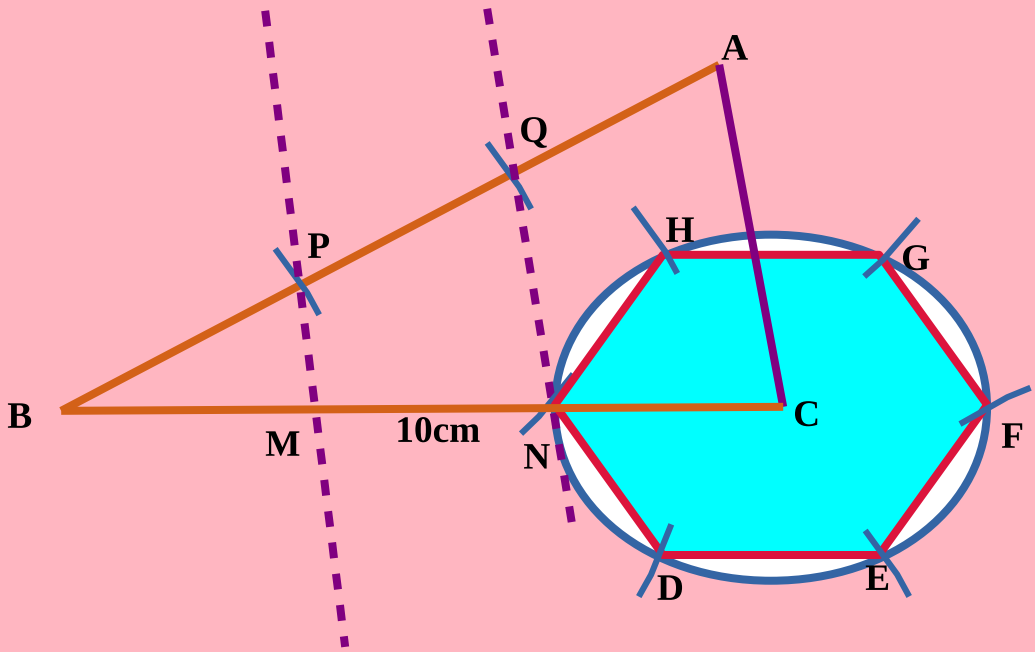
Draw lines parallel to QR from A to PQ. Then using the 2 equal parts obtained in PQ take PS or SQ as side and draw square SQTU.



Here SQ+QT+TU+US=15cm

(3) *Draw a regular hexagon of perimeter 20 centimetres.*

ANSWER



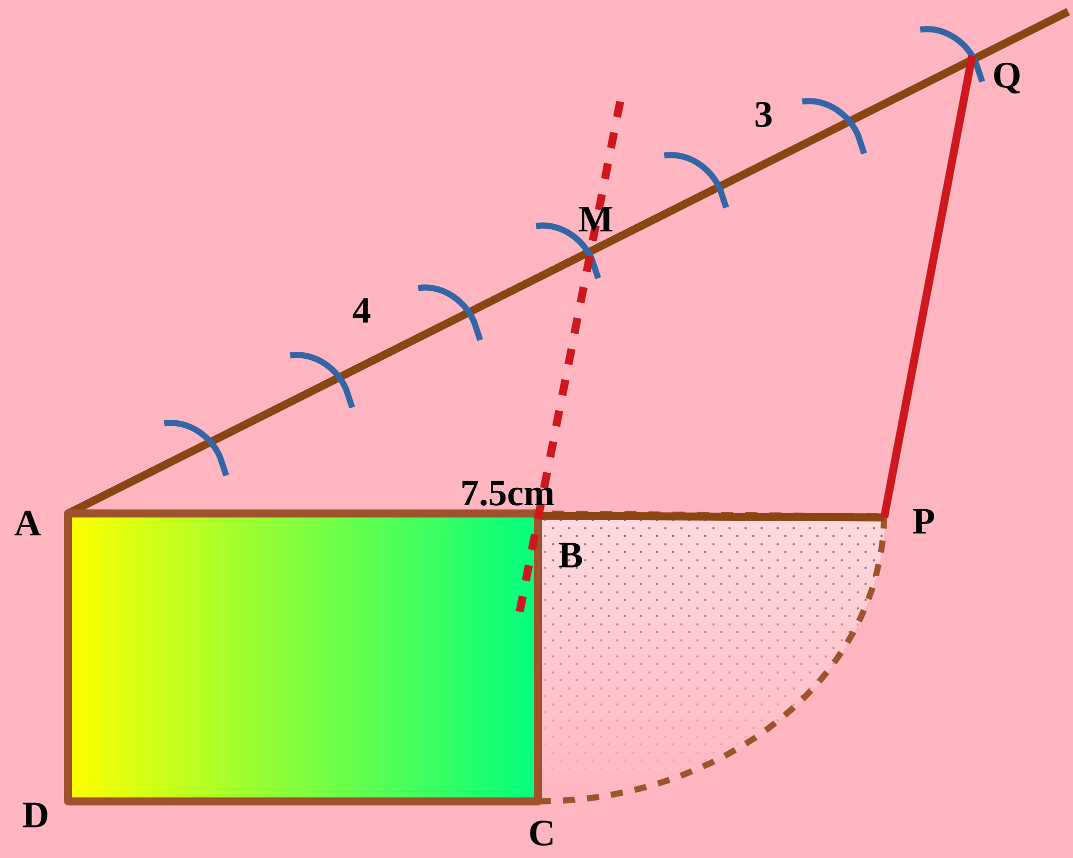
- *Draw a line BC of length 10cm. From B to A , 3 equal parts mark points P and Q with compass and join A and C .*
- *Draw lines parallel to AC from P and Q to BC.*
- *Then from the 3 equal parts obtained in BC, take NC as radius and draw a circle with C as center.*
- *Take NC as radius and divide the circle into 6 equal parts and draw a regular hexagon NDEFGH.*

Three or more parallel lines cut any two lines in the same ratio.

Page No: 46

(1) Draw a rectangle of perimeter 15 centimetres and sides in the ratio 3: 4.

ANSWER



Perimeter is 15 cm, so length + breadth = 7.5 cm

- *Draw AP, a line segment of length 7.5 cm.*
- *Draw AQ from A and mark 7 equal units on it*
- *Mark M on this line such that AM = 4 units, and join PQ.*
- *Draw a line parallel to PQ and passing through M, which meets AP at B.*
- *Draw rectangle ABCD whose length as AB and breadth as BP*

(2) Draw a triangle of each of the types below, of perimeter 13 centimetres:

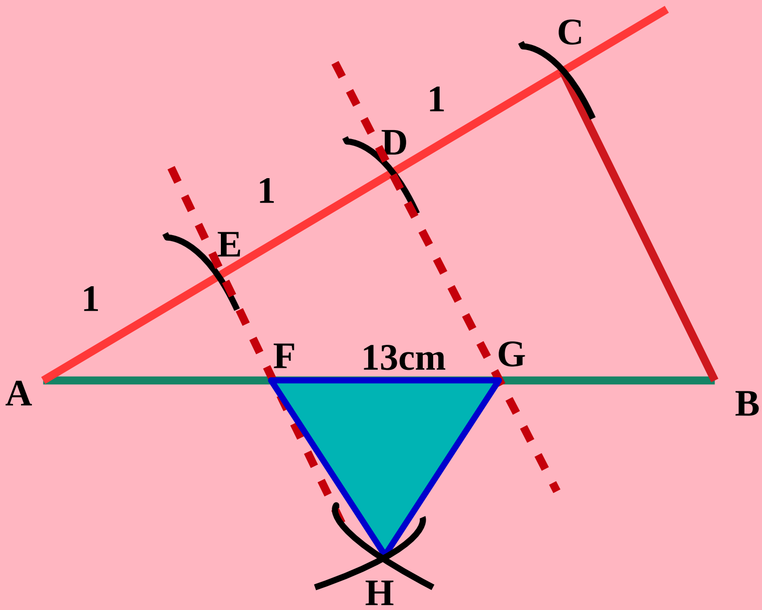
(i) Equilateral

(ii) Sides in the ratio 3 : 4 : 5

(iii) Isosceles with lateral sides one and a half times the base

ANSWER

(i).

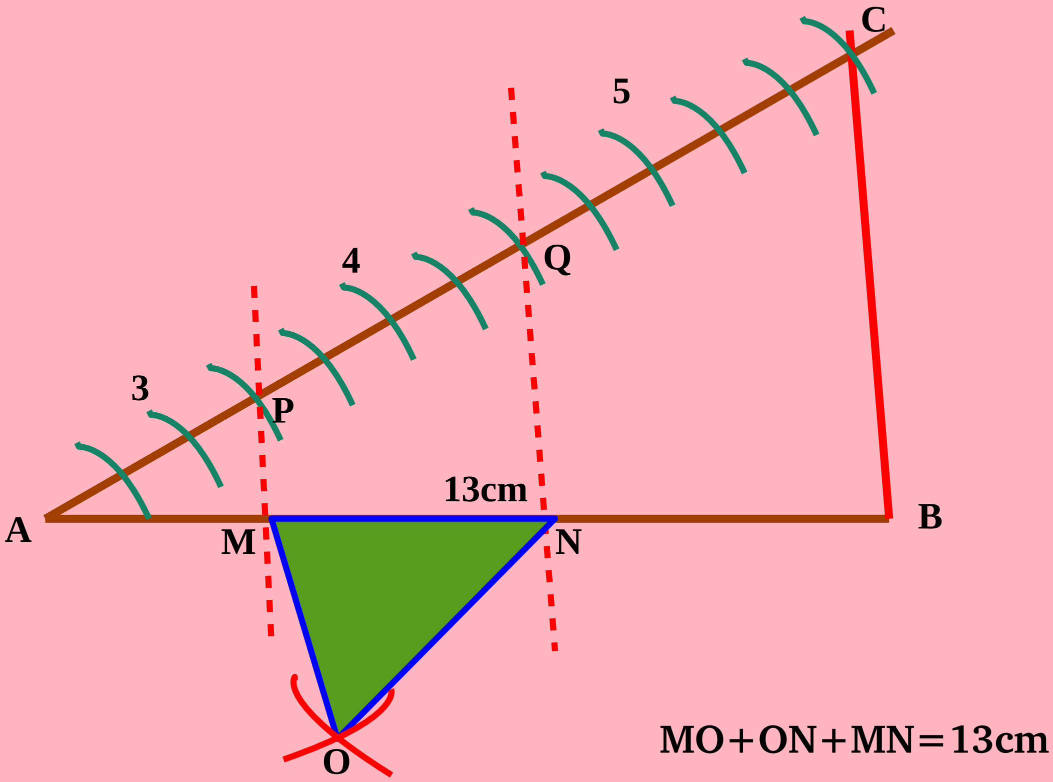


FH+FG+GH=13cm

- *Draw a line AB of 13 cm*
- *Draw AC from A and mark 3 equal units on it*
- *Mark E ,D on this line AC such that AE = ED=DC= 1 unit, and join BC.*
- *Draw a line parallel to BC and passing through D and E,*

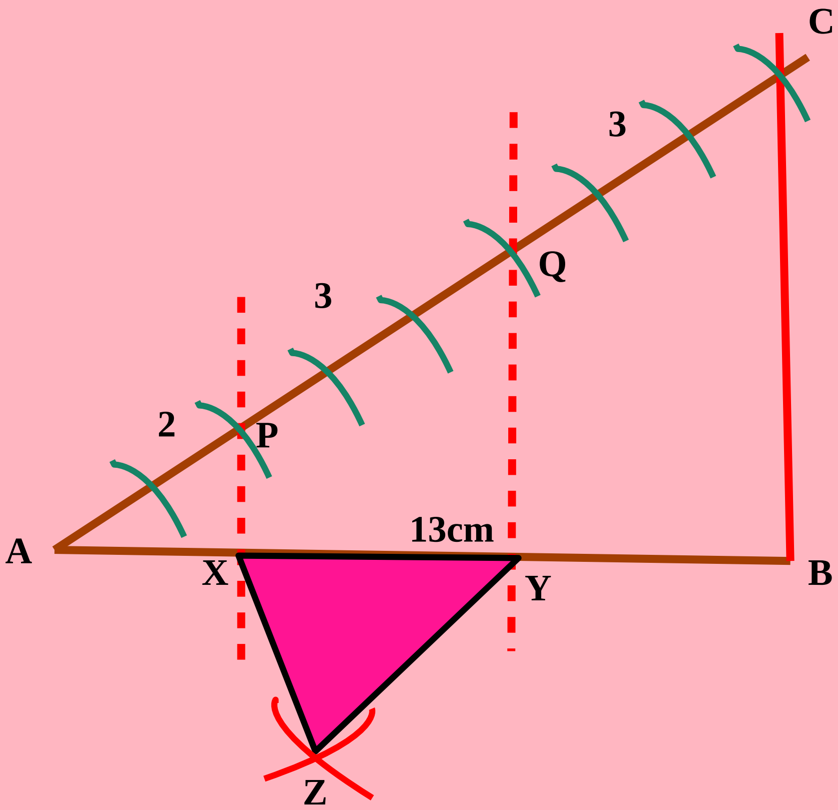
  which meets AB at G and F.
- *Draw an equilateral triangle FGH whose length taken as FG or GB or AF*

(ii)



- Draw a line AB of 13 *cm* .
- *Draw AC from A and mark 12 equal units on it*
- *Mark P ,Q on this line AC such that AP = 3 units, PQ=4 units and QC= 5 units. Join BC.*
- *Draw a line parallel to BC and passing through P and Q ,* which meets AB at M and N.
- *Construct* △MNO *with lengths AM, NB as sides using compass and* MN *as the base.*

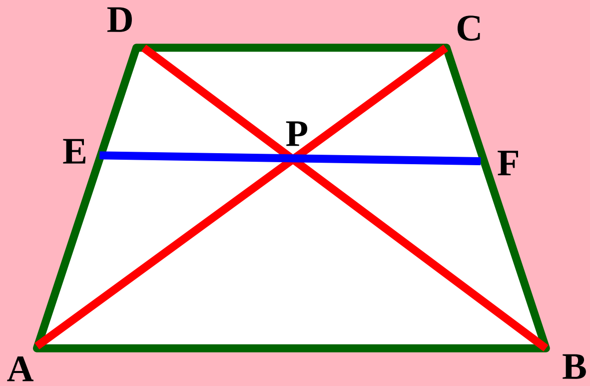
(iii).



- Draw a line AB of 13*cm* .
- *Draw AC from A and mark 8 equal units on it*
- *Mark P ,Q on this line AC such that* AP = *2 units,* PQ=QC=*3 units and Join BC.*
- *Draw a line parallel to BC and passing through P and Q ,* which meets AB at X and Y.
- *Construct* △XYZ *with lengths* AX, YB *as sides using compass and* XY *as the base.*

(3) Prove that in any trapezium, the diagonals cut each other in the same ratio.

ANSWER



AB *and* DC *are parallel.* A *line* EF *is drawn parallel to it.*
Three parallel lines intersect lines AC *and* BD *in the same ratio.*

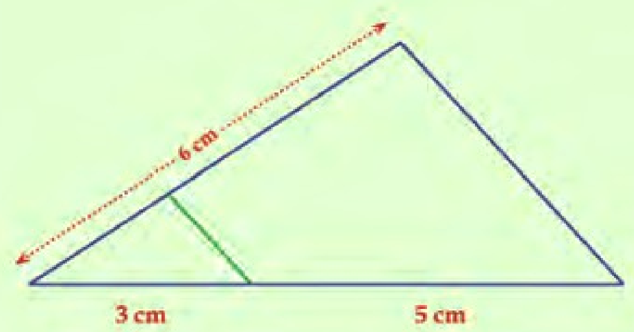
$\therefore \frac{PD}{PB} = \frac{PC}{PA}$

Page No: 50 & 51

In any triangle, a line parallel to one side cuts the other two sides into the same parts.

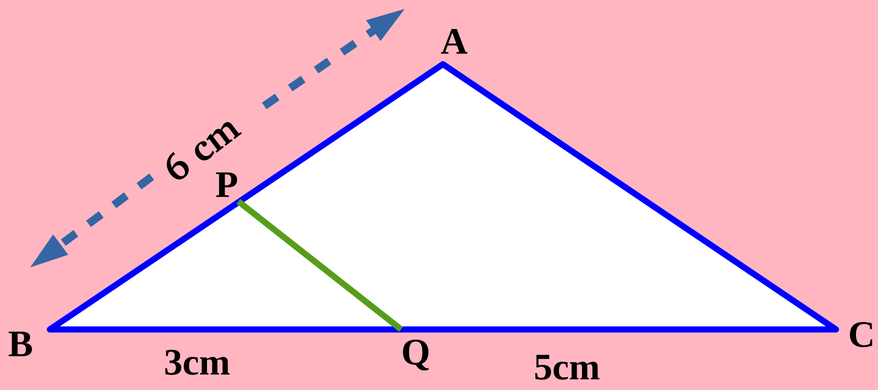
In any triangle, a line dividing two sides in the same ratio is parallel to the third side.

(1) In the picture below, the green line is parallel to the right side of the blue triangle.



Calculate the lengths of the pieces into which this line cuts the left side.

ANSWER



Since PQ is parallel to AC

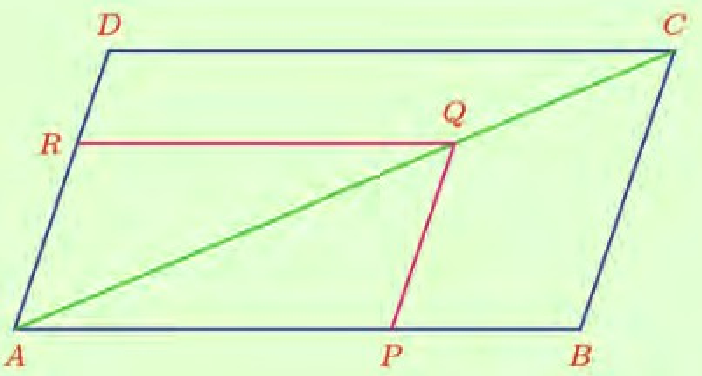
$$\frac{BQ}{BC} = \frac{BP}{BA}$$

$$\frac{3}{8} = \frac{BP}{6}$$

BP $= \frac{3 \times 6}{8} = \frac{18}{8} = \frac{9}{4} = 2.25$ **cm**

AP $= 6 - 2.25 = 3.75$**cm**

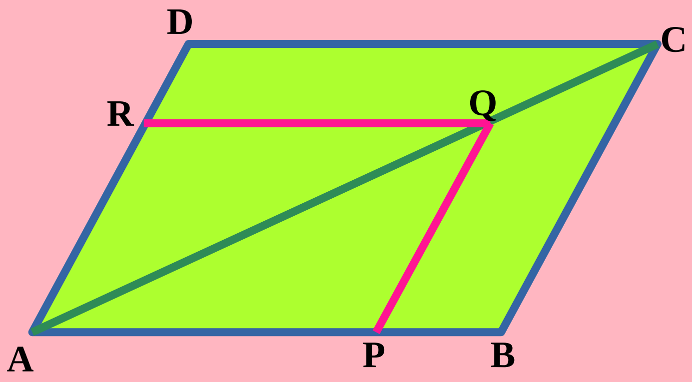
(2) In the parallelogram $ABCD$, the line through the point P on AB, parallel to BC meets AC at Q. The line through Q parallel to AB meets AD at R:



(i) Prove that $\frac{AP}{PB} = \frac{AR}{RD}$

(ii) Prove that $\frac{AP}{AB} = \frac{AR}{AD}$

ANSWER



(i). ***In △ABC***

$$\frac{AP}{PB} = \frac{AQ}{QC} \text{ --------- } (1)$$

In △ADC

$$\frac{AQ}{QC} = \frac{AR}{RD} \text{ --------- } (2)$$

From (1) and (2)

$$\frac{AP}{PB} = \frac{AR}{RD}$$

(ii). *From* $\frac{AP}{PB} = \frac{AR}{RD}$

$$\frac{PB}{AP} = \frac{RD}{AR}$$

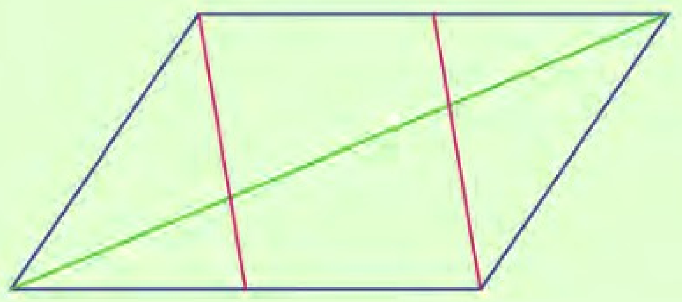
$$\frac{PB}{AP} + 1 = \frac{RD}{AR} + 1$$

$$\frac{PB+AP}{AP} = \frac{RD+AR}{AR}$$

$$\frac{AB}{AP} = \frac{AD}{AR}$$

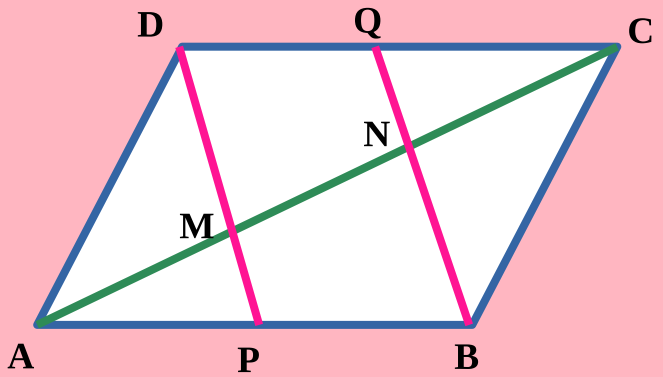
$$\frac{AP}{AB} = \frac{AR}{AD}$$

(3) In the picture below, two corners of a parallelogram are joined to the midpoints of two sides.



Prove that these lines cut the diagonal in the picture into three equal parts.

ANSWER



In the figure ABCD *is a parallelogram.*

AB = CD, AB || CD.

P, Q are midpoints of sides AB and DC respectively.

PB = $\frac{1}{2}$ AB

DQ = $\frac{1}{2}$ CD = $\frac{1}{2}$ AB (AB=CD)

∴ PB = DQ

That is, PB = DQ *and* PB || DQ *also* PD || BQ

∴ *PBQD is also parallelogram.*

In △ABN

$$\frac{AM}{MN} = \frac{AP}{PB}$$

$$\frac{AM}{MN} = 1$$

AM = MN ------------*(1)*

In △CDM

$$\frac{CN}{MN} = \frac{CQ}{DQ}$$

$$\frac{CN}{MN} = 1$$

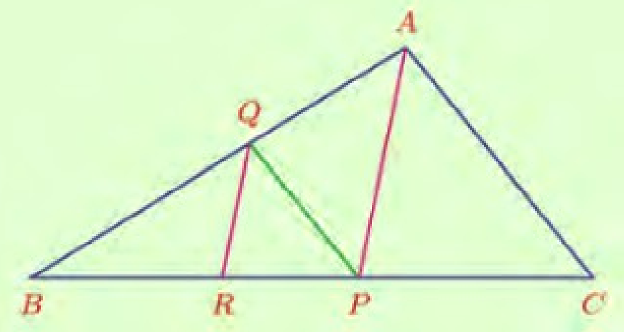
CN = MN ------------*(2)*

From (1) and (2)

CN = MN =AM

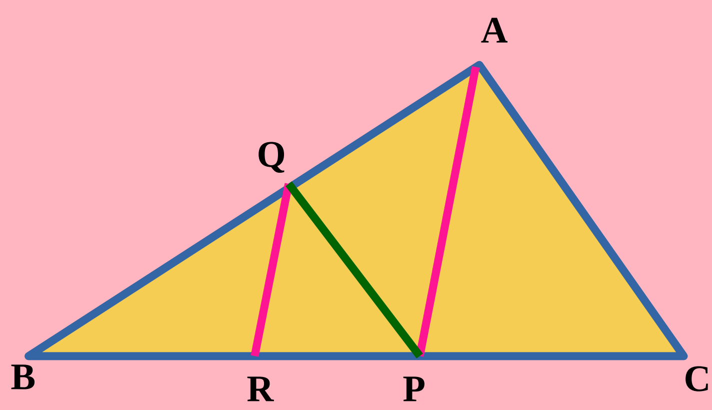
It is proved that the diagonal AC *is divided into three equal parts.*

(4) In triangle ABC, the line parallel to AC through the point P on BC, meets AB at Q. The line parallel to AP through Q meets BC at R:



Prove that $\frac{BP}{PC} = \frac{BR}{RP}$

ANSWER



We know that , **QR‖ PA , QP‖ AC**

In **△ABC**

$\frac{BP}{PC} = \frac{BQ}{QA}$ ***--------- (1)***

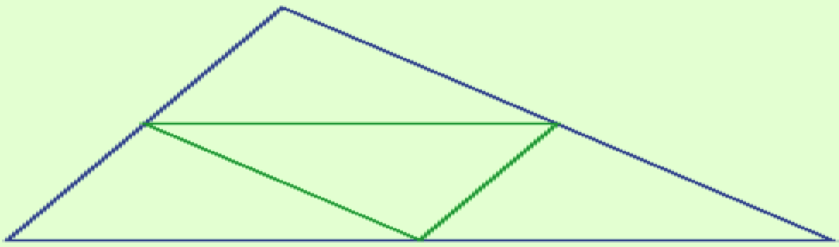
In **△BPA**

$\frac{BR}{RP} = \frac{BQ}{QA}$ ***--------- (2)***

From (1) and (2) $\frac{BP}{PC} = \frac{BR}{RP}$

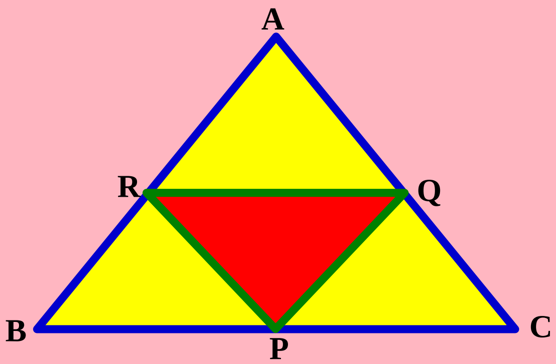
Page No: 53,54 & 55

(1) In the picture, the midpoints of a triangle are joined to form a smaller triangle inside:



How many times the perimeter of the small triangle is the perimeter of the large triangle? What about their areas?

ANSWER



(i) *Since* P, Q, R *are midpoints*

AC= 2PR --------*(1)*

AB= 2PQ --------*(2)*

BC= 2RQ --------*(3)*

Perimeter of △ABC = AB + BC + AC= 2PQ + 2QR+ 2PR

Perimeter of △ABC = 2 (PQ + QR + PR)

Perimeter of △ABC= 2 × Perimeter of △PQR

∴ The perimeter of the larger triangle is 2 times the perimeter of the smaller triangle.

(ii).

Since all four triangles are congruent triangles.

∴The area of the larger triangle 4 times the area of the smaller triangle.

(2) See these pictures:

The first picture is that of a triangle cut out from a sheet of paper. The second one shows it with the small triangle in the middle, joining the midpoints of the sides, cut off from the large triangle.

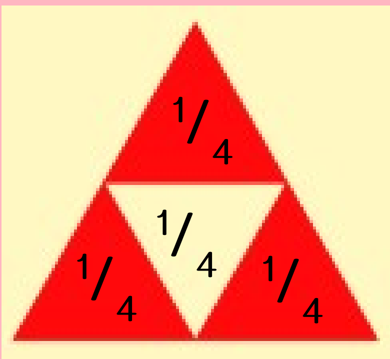
The third picture shows such middle pieces cut off from each of the small triangles in the second picture

(i) What fraction of the area of the paper in the first picture is the area of the paper in the second picture?

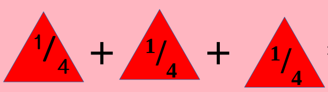
(ii) What about in the third picture?

ANSWER

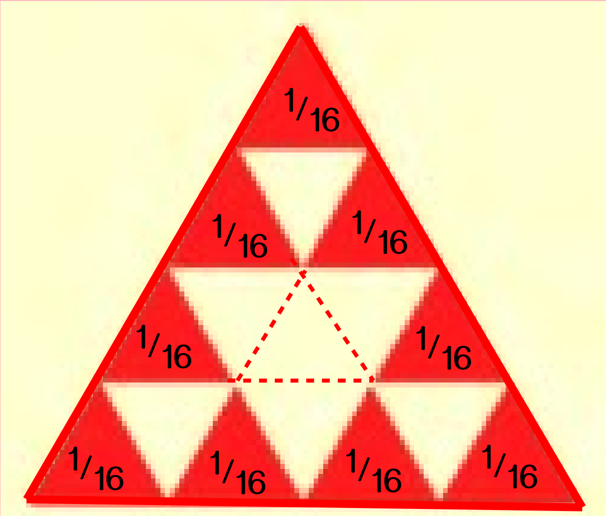
(i).



The area of the paper in the second picture is of the $\frac{3}{4}$ *parts area in the first picture* $\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} \right)$

 $= \frac{3}{4}$

(ii).



That is, each small triangle in the third figure is $\frac{1}{4}$ *of* $\frac{1}{4}$ *of the first triangle.* ($\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ *part*) .

Hence the area of the paper in the third figure

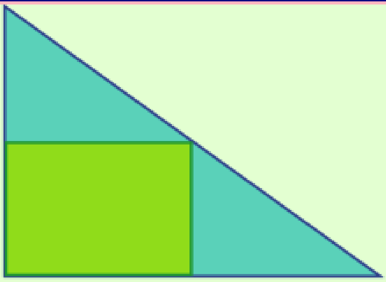
$$= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$$

$= \frac{9}{16}$ *part* .

(3) A quadrilateral is drawn with the midpoints of the sides of a right triangle and its square corner as vertices.

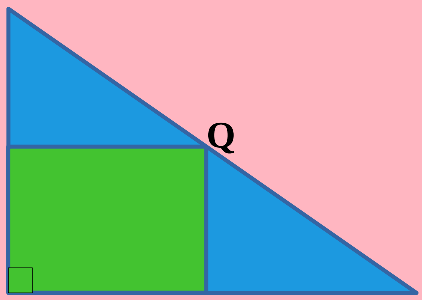
(i) Prove that this quadrilateral is a rectangle.

(ii) What fraction of the area of the triangle is the area of the rectangle?



ANSWER

(i).



P *and* R *are the midpoints of the sides* AB *and* BC *and* $\angle$ B=90°

PQ= $\frac{1}{2}$ BC

PQ=BR

PQ ∥ BC

PQ ∥ BR

Quadrilateral PQRB *is a parallelogram.*

Since $\angle$ B=90°

$\angle$ P= $\angle$ Q= $\angle$ R = 90°

∴ PQRB *is a rectangle.*

(ii). *Area of rectangle* = BR × PB

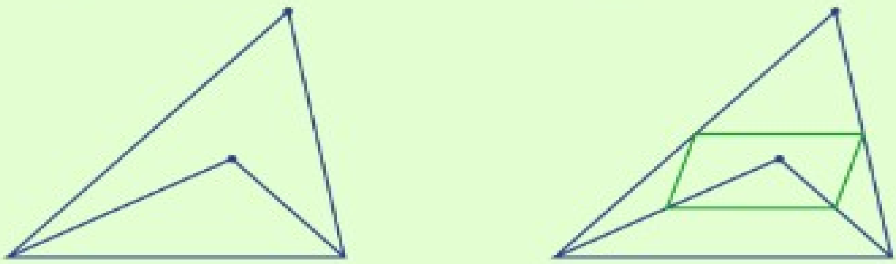
$$= \frac{1}{2} BC \times \frac{1}{2} AB$$

$$= \frac{1}{2} \times \left(\frac{1}{2} \times BC \times AB \right)$$

$$= \frac{1}{2} \times \text{Area of } \triangle ABC$$

∴ *The area of the rectangle is half the area of the triangle.*

(4) In the two pictures below, the first one shows two triangles formed by joining the ends of a line to two points above it. The second one shows the quadrilateral formed by joining the midpoints of the left and right sides of these triangles:



(i) Prove that this quadrilateral is a parallelogram.

(ii) Describe the positions of the points on top for this quadrilateral to be:

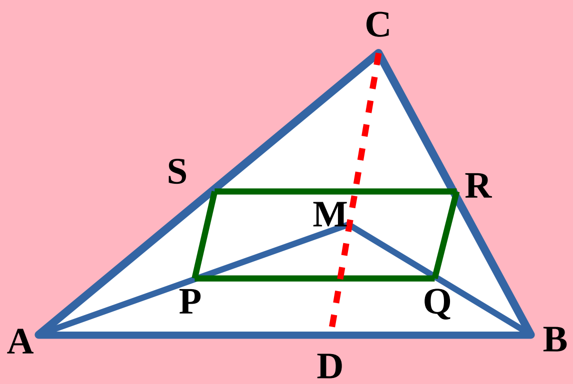
(a) Rhombus (b) Rectangle

(c) Square

(iii) Would we get all these, if one point is taken above the line and one below?

ANSWER

(i)



In $\triangle$ABM , PQ ∥ AB (P,Q *are midpoints of* AM, BM *respectively*)

PQ = $\frac{1}{2}$ AB -------(1)

In $\triangle$ABC , SR ∥ AB (S, R *are midpoints of* AC, BC *respectively*)

SR = $\frac{1}{2}$ AB -------(2)

From (1) *and* (2)

PQ = SR

∴ PQRS *is a parallelogram.*

ii)

a) Wherever CM = AB *we get a rhombus.*

b) *Angles must be 90° and* RS, SP *must be perpendicular to form a rectangle,* AB *and* CD *parallel to these should be perpendicular, That is, their position should be on a line perpendicular to* AB.

c) To be a square, the sides must be equal and the angles must be 90°.

Here is rectangle and rhombus.

That is, points C *and* M *should be on a line perpendicular to* AB, *and* AB = CM

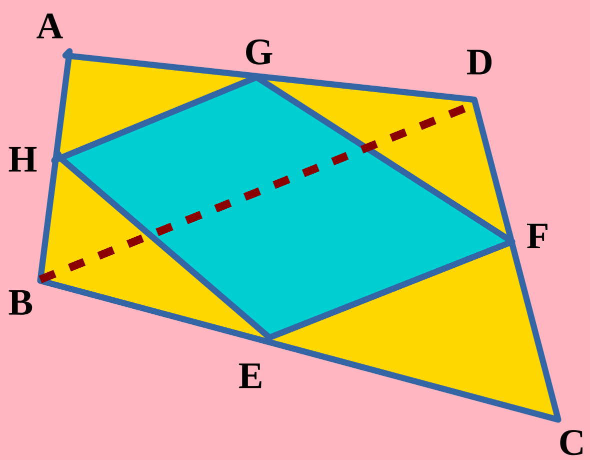
iii) *Even if the point is on both sides, the same things will happen.*

(5) (i) Prove that the quadrilateral formed by joining the midpoints of any quadrilateral is a parallelogram.

(ii) Explain what should be the original quadrilateral to get the inner quadrilateral as

(a) Rhombus (b) Rectangle (c) Square

ANSWER



(i).

If E,F,G,H *are the midpoints of the sides BC, CD, AD and AB respectively*

In △BAC , GH = $\frac{1}{2}$ BD , GH ǁ BD ---------*(1)*

In △BCD , EF = $\frac{1}{2}$ BD , EF ǁ BD ---------*(2)*

From (1) and (2)

We get GH = EF *and* GH ǁ EF .

∴ EFGH *is a parallelogram.*

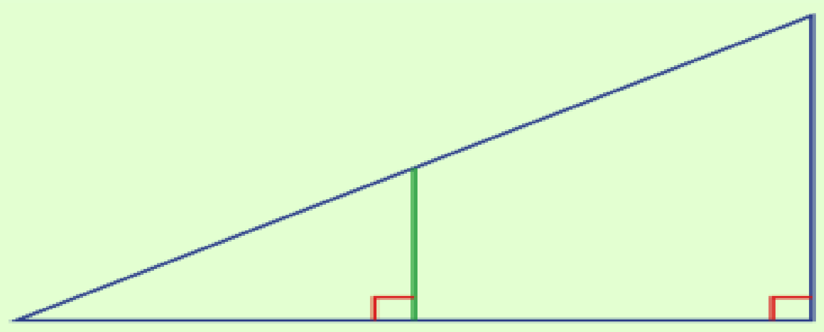
ii) a) Rhombus - A quadrilateral with equal diagonals

b) Rectangle - A quadrilateral with vertical diagonals .

c) Square - A quadrilateral with equal and vertical diagonals .

Page No: 58, 59& 60

(1) Draw a right triangle and draw the perpendicular from the midpoint of the hypotenuse to the base:

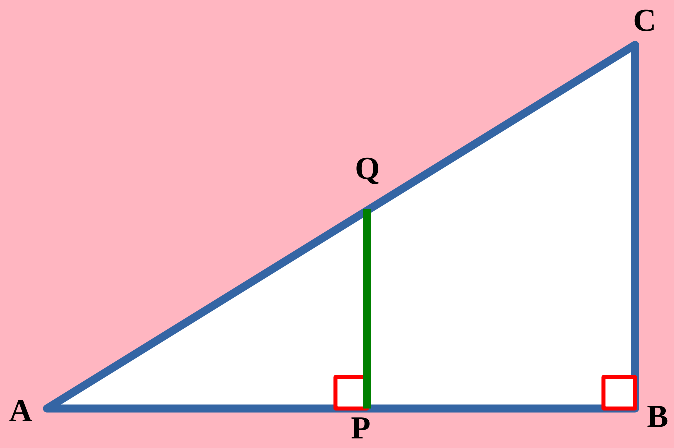


(i) Prove that this perpendicular is half the vertical side of the large triangle.

(ii) Prove that the distances from the midpoint of the hypotenuse to the three vertices of the large triangle are equal.

(iii) Prove that the circumcentre of a right triangle is the midpoint of its hypotenuse.

ANSWER



(i). $\angle P = \angle B = 90^\circ$

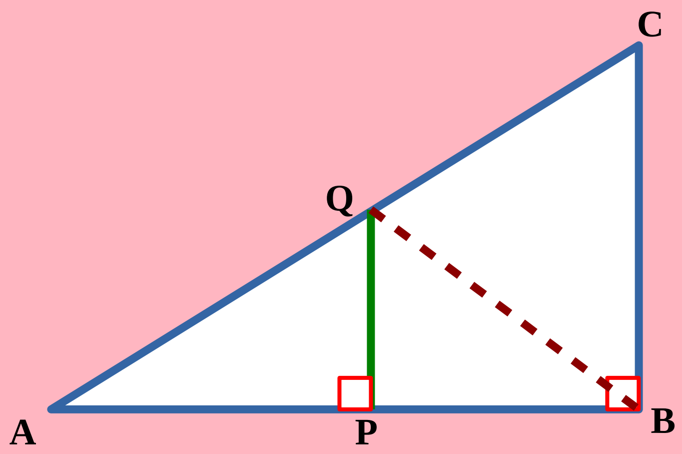
$\therefore$ PQ || BC

And Q is the midpoint of AC.

$\therefore$ P is the midpoint of AB.

$\therefore PQ = \frac{1}{2} BC$

(ii).



Q is the midpoint of AC.

∴ QA = QC --------(1)

Consider △APQ and △BPQ

AP= PB (P *is the midpoint of* AB.)

PQ = PQ *(Common side)*

∠APQ = ∠BPQ *(90°)*

△APQ *and* △BPQ *are congruent triangles.*

∴QA = QB --------(2)

From (1) *and* (2)

∴ QA = QB = QC

∴ *It is proved that the distance from the midpoint of the hypotenuse to the three vertieces of the large triangle are equal.*

(iii). QA = QB = QC

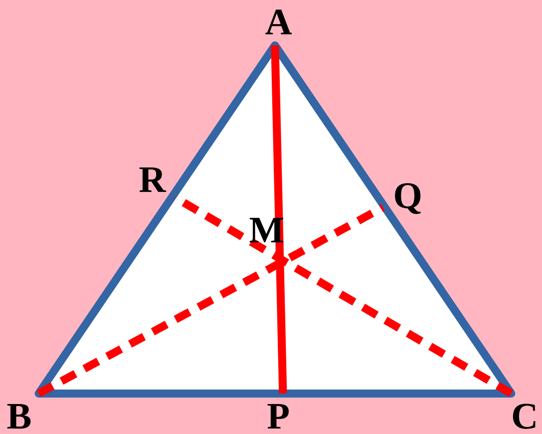
A circle with center Q *and radius* QA *is drawn through the points* A, B *and* C *will pass.*

That is, this circle is the circumcircle of △ABC.

∴ *The circumcenter of a right triangle is the midpoint of its hypotenuse.*

(2) Prove that in any equilateral triangle, the circumcentre, orthocentre and the centroid coincide.

ANSWER



△ABC *is an equilateral triangle.*

Draw a perpendicular from A *to* BC.

$\angle APB = \angle APC = 90^\circ$

That means AP *is height, and* BP = CP

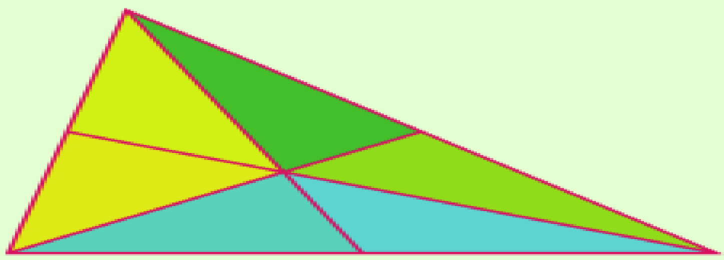
A point P *bisects* BC. *That means* AP *is the median.*

The line AP *is perpendicular and bisector of* BC, *so* AP *is perpendicular bisector, So are the lines* BQ *and* CR.

Lines AP, BQ *and* CR *meet at point* M

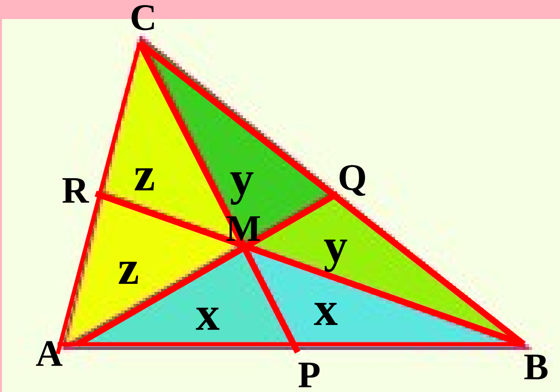
∴ *For any equilateral triangle, the circumcenter , orthocentre and the centroid coincide*

(3) In the picture below, the medians of the triangle divide it into six small triangles:



Prove that all six triangles have the same area.

ANSWER



Area of $\triangle$APM $=$ *Area of* $\triangle$BPM $= x$ (*Because MP is median*)

Area of $\triangle$BQM $=$ *Area of* $\triangle$CQM $= y$ (*Because MQ is median*)

Area of $\triangle$CRM $=$ *Area of* $\triangle$ARM $= z$ (*Because MR is median*)

If AP is median

$2z + x = 2y + x$

$z = y$ ------(1)

If BR is median,

$2y + z = 2x + z$

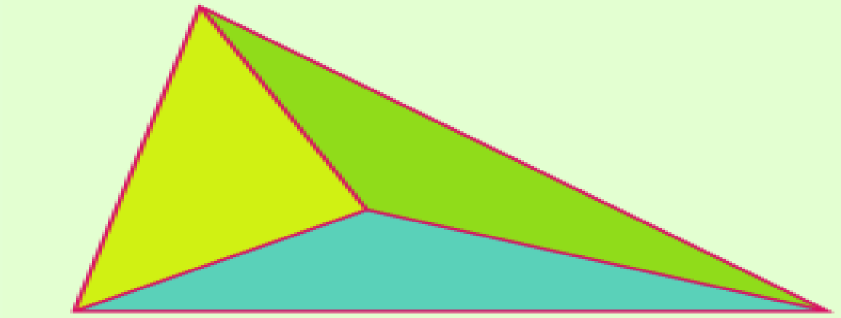
$y = x$ ------(2)

From (1) and (2)

$x = y = z$

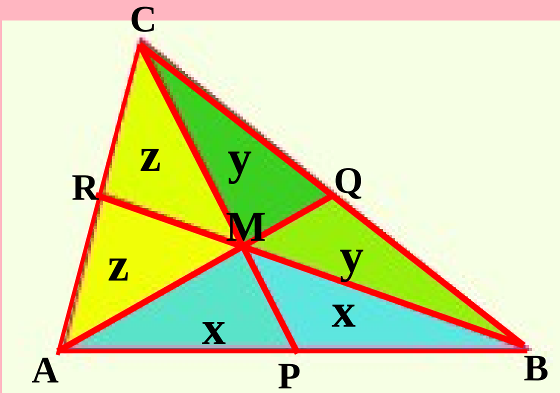
$\therefore$ *All six triangles have the same area.*

(4) In the picture below, the triangle is divided into three small triangles by joining the centroid to the three vertices:



Prove that all three triangles have the same area.

ANSWER



Area of $\triangle$APM = *Area of* $\triangle$BPM = *x* (*Because MP is median*)

Area of $\triangle$BQM = *Area of* $\triangle$CQM = *y* (*Because MQ is median*)

Area of $\triangle$CRM = Area of $\triangle$ARM = z (Because MR is median)

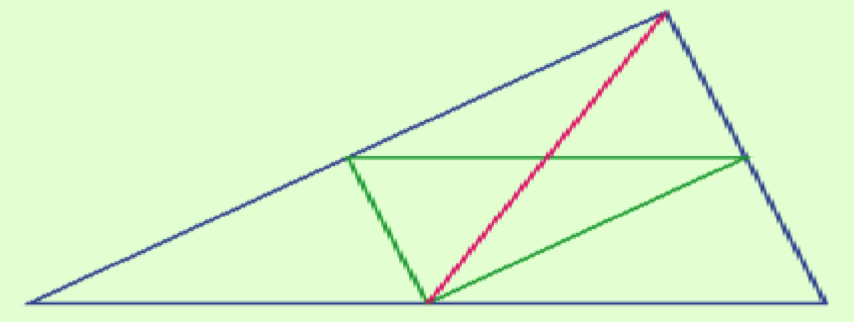
Since $x = y = z$

$2x = 2y = 2z$

$\therefore$ *Area of* $\triangle$ABM = *Area of* $\triangle$BMC = *Area of* $\triangle$AMC.

$\therefore$ *All three triangles have the same area.*

(5) In the picture below, the midpoints of the sides of the blue triangle are joined to make the smaller green triangle. The red line is a median of the large triangle.

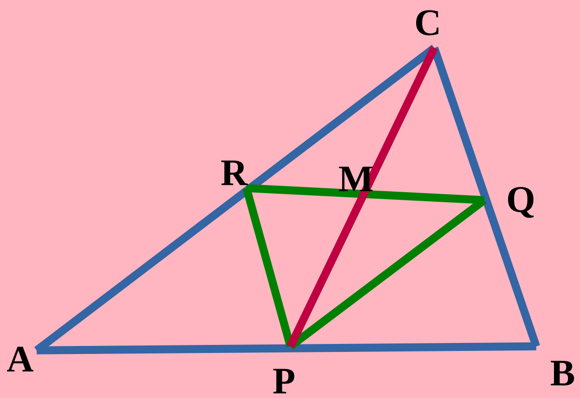


(i) Prove that this median bisects the top side of the small triangle.

(ii) Prove that the centroid of the large and small triangles are the same.

ANSWER

(i).

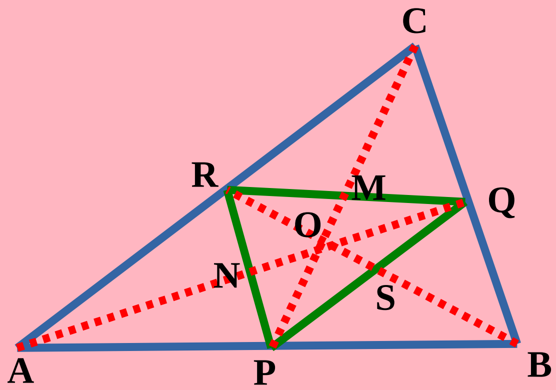


The third line joining the midpoints of two sides of a triangle is parallel to the third side

∴ PQCR *is a parallelogram.*

Since the diagonals of the parallelogram are equal to each other, RM=QM.

(ii).



Let O *be the centroid of* $\triangle$ABC

RM = QM

RN = PN

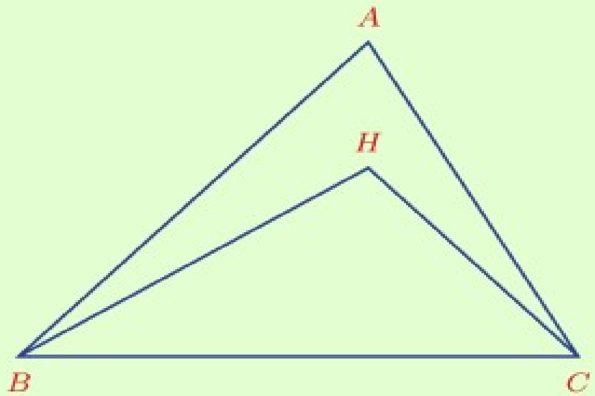
PS = QS

That is, RS, QN *and* PM *are medians.*

$\therefore$ *The centroid of* $\triangle$PQR *is* O.

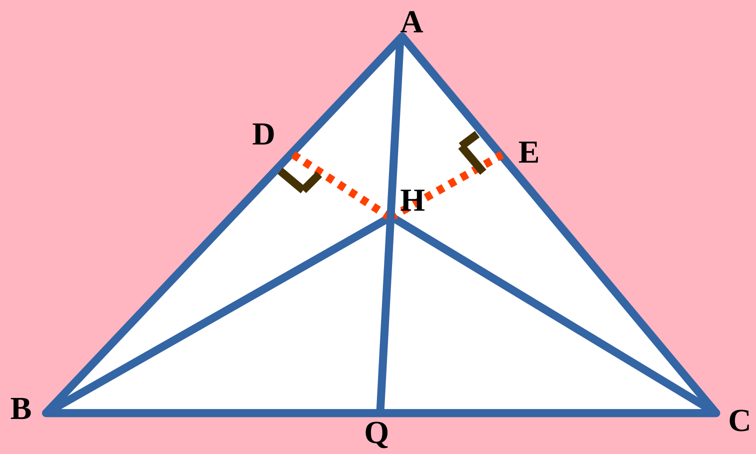
$\therefore$ *The centroid of the large and small triangles are the same..*

(6) In the picture below, *H* is the orthocentre of triangle *ABC*.



Prove that *A* is the orthocentre of triangle *HBC*

ANSWER



H *is the orthocentre of* △ ABC

BE *is perpendicular to* AC

AC *is the perpendicular from* C *to the side* BH

Since CD *is the perpendicular* AB, AB *is the perpendicular from B to the side* CH.

The point where these intersect at A

∴ A *is the orthocentre of triangle* HBC