MULTIPLICATION IDENTITIES

4

To multiply a sum by a sum, we must multiply each number in the first sum by each number in the second sum and add all these products together.

For any positive numbers x, y, u, v

$$(x + y)(u + v) = xu + xv + yu + yv$$

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(1)   Mentally find the products below:

(i) 71×91 (ii) 42×62 (iii) $10\frac{1}{2} \times 6\frac{1}{2}$ (iv) 9.5×3.5 (v) $10\frac{1}{4} \times 6\frac{1}{4}$

ANSWER

(i) 71×91

$(x+1)(y+1) = xy+x+y+1$

$(70+1)(90+1) = 70 \times 90 + 70 + 90 + 1$

$= 6300 + 160 + 1$

$= 6461$

(ii) 42×62

$(x+2)(y+2) = xy + 2(x+y) + 4$

$(40+2)(60+2) = 40 \times 60 + 2 \times (40+60) + 4$

$= 2400 + 200 + 4$

$= 2604$

(iii) $10\frac{1}{2} \times 6\frac{1}{2}$

$$\left(x + \frac{1}{2}\right)\left(y + \frac{1}{2}\right) = xy + \frac{1}{2}(x+y) + \frac{1}{4}$$

$$\left(10 + \frac{1}{2}\right)\left(6 + \frac{1}{2}\right) = 10 \times 6 + \frac{1}{2}(10+6) + \frac{1}{4}$$

$$= 60 + \frac{1}{2} \times 16 + \frac{1}{4}$$

$$= 60 + 8 + \frac{1}{4}$$

$$= 68 + \frac{1}{4}$$

$$= 68\frac{1}{4}$$

(iv) 9.5×3.5

$$9.5 \times 3.5 = (9 + 0.5)(3 + 0.5)$$

$$= \left(9 + \frac{1}{2}\right)\left(3 + \frac{1}{2}\right)$$

$$= 9 \times 3 + \frac{1}{2}(9+3) + \frac{1}{4}$$

$$= 27 + \frac{1}{2} \times 12 + \frac{1}{4}$$

$$= 27 + 6 + \frac{1}{4}$$

$$= 33\frac{1}{4}$$

$$= 33.25$$

(v) $10\frac{1}{4} \times 6\frac{1}{4}$

$$10\frac{1}{4} \times 6\frac{1}{4} = \left(10 + \frac{1}{4}\right)\left(6 + \frac{1}{4}\right)$$

$$= 10 \times 6 + \frac{1}{4}(10 + 6) + \frac{1}{16}$$

$$= 60 + \frac{1}{4} \times 16 + \frac{1}{16}$$

$$= 60 + 4 + \frac{1}{16}$$

$$= 64\frac{1}{16}$$

(2) The product of two numbers is 1400 and their sum is 81. What is the product of the numbers next to each?

ANSWER

Two numbers are x, y

Next numbers to each are $(x+1)$, $(y+1)$

$xy = 1400$

$x+y = 81$

$(x+1)(y+1) = xy+x+y+1$

The product of next numbers to each are $= 1400 + 81 + 1 = 1482$

(3) The product of two odd numbers is 621 and their sum is 50. What is the product of the odd numbers next to each?

ANSWER

Odd numbers are x, y

Next odd numbers to each are $(x+2)(y+2)$

$xy = 621$

$x+y = 50$

$(x+2)(y+2) = xy + 2(x+y)+4$

$\qquad = 621 + 2 \times 50 + 4$

$\qquad = 621 + 100 + 4 = 725$

The product of next odd numbers to each are $= 725$

(1) Calculate each of the following for several numbers and guess a generalization; then prove it using algebra.

   (i) The remainder got on division by 3, the product of two numbers one of which leaves remainder 1 on division by 3 and the other, remainder 2.

   (ii) The remainder got on division by 4, the product of two numbers one of which leaves remainder 1 on division by 4 and the other, remainder 2.

   (iii) The difference of the products of the two numbers at the ends and the two in the middle, of six consecutive natural numbers.

ANSWER

   (i)   A number with a remainder of 1 and a number with a remainder of 2 when divided by 3 ,the remainder after dividing the product by 3 is

$$(3x+1)(3x+2) = 9x^2 + 6x + 3x + 2$$

$$= 9x^2 + 9x + 2$$

$$= 3(3x^2 + 3x) + 2$$

∴ Divide by 3 and the remainder is 2

   (ii) A number with a remainder of 1 and a number with a remainder of 2 when divided by 4 ,the remainder after dividing the product by 4 is

$$(4x+1)(4x+2) = 16x^2 + 8x + 4x + 2$$

$$= 16x^2 + 12x + 2$$

$$= 4(4x^2 + 3x) + 2$$

∴ Divide by 4 and the remainder is 2

(iii) Consicutive natural numbers are $x, x+1, x+2, x+3, x+4, x+5$

The product of two numbers at the end$= x(x+5) = x^2+5x$

The product of two numbers in the midle $= (x+2)(x+3)$

$$= x^2+3x+2x+6$$

$$= x^2+5x+6$$

∴ The difference between these $= (x^2+5x+6) - (x^2+5x) = 6$

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(2) Given below is a method to find the product 36 × 28

3 × 2 = 6	6 × 100	600
(3 × 8) + (6 × 2) = 36	36 × 10	360
6 × 8		48
————————————		————
36 × 28		1008

(i) Check these for some other products of two digit numbers.

(ii) Explain why this works using algebra.

(Recall the general algebraic form 10*m* + *n* of a two digit number, seen in Class 7)

ANSWER

(i) 43 × 26

4 × 2	=	8	8 × 100	= 800
4 × 6 + 3 × 2	=	30	30 × 10	= 300
3 × 6	=	18		= 18
				= 1118

(ii) One number be $(10x+y)$ and other be $(10m+n)$

Then their product$= (10x+y)(10m+n)$

$(10x+y)(10m+n) = (10x \times 10m) + (10x \times n) + (y \times 10m) + (y \times n)$

$$= 100xm + 10xn + 10ym + yn$$

$$= 100xm + 10(xn + ym) + yn$$

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(1) Write numbers as shown below:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

(i) As we did in the calendar, mark a square of four numbers and find the difference of the diagonal products. Do we get the same difference for any such square?

(ii) Explain why this is so, using algebra.

ANSWER

(i)

2	3
7	8

$2 \times 8 = 16$

$7 \times 3 = 21$

Difference $= 21 - 16 = 5$

6	7
11	12

$6 \times 12 = 72$

$7 \times 11 = 77$

Difference $= 77 - 72 = 5$

∴ The difference is the same

(ii)

x	$x+1$
$x+5$	$x+6$

$x(x+6) = x^2 + 6x$

$(x+5)(x+1) = x^2 + x + 5x + 5$

$\qquad\qquad\quad = x^2 + 6x + 5$

∴ **Difference** $= (x^2 + 6x + 5) - (x^2 + 6x) = 5$

(2) In the multiplication table we've made, draw a square of nine numbers, instead of four, and mark the numbers at the four corners:

⑥	8	⑩
9	12	15
⑫	16	⑳

(i) What is the difference of the diagonal sums ?

(ii) Explain using algebra why we get the difference as 4 in all such squares.

(iii) What about a square of sixteen numbers?

ANSWER

(i)

6	8	10
9	12	15
12	16	20

$6 + 20 \ = 26$

$12 + 10 = 22$

∴ **Difference** $= 26 - 22 \ = 4$

(ii)

xy	$x(y+1)$	$x(y+2)$
$(x+1)y$	$(x+1)(y+1)$	$(x+1)(y+2)$
$(x+2)y$	$(x+2)(y+1)$	$(x+2)(y+2)$

$$xy+(x+2)(y+2) = xy+ xy+2x+2y+4$$
$$= 2xy+2x+2y+4 \text{ --------(1)}$$
$$x(y+2)+(x+2)y = xy+2x+xy+2y$$
$$= 2xy + 2x +2y \text{ --------(2)}$$
$$(1)- (2) = (2xy+2x+2y+4) - (2xy + 2x +2y) = 4$$
$$\therefore \text{Difference} = 4$$

(iii)

6	8	10	12
9	12	15	18
12	16	20	24
15	20	25	30

$$6 + 30 = 36$$
$$12 + 15 = 27$$
$$\therefore \text{Difference} =36 - 27 = 9$$

xy	$x(y+1)$	$x(y+2)$	$x(y+3)$
$(x+1)y$	$(x+1)(y+1)$	$(x+1)(y+2)$	$(x+1)(y+3)$
$(x+2)y$	$(x+2)(y+1)$	$(x+2)(y+2)$	$(x+2)(y+3)$
$(x+3)y$	$(x+3)(y+1)$	$(x+3)(y+2)$	$(x+3)(y+3)$

$$xy+(x+3)(y+3) = xy+ xy+3x+3y+9 \text{ --------(1)}$$

$$(x+3)y+x(y+3) = xy+3y+xy+3x \text{ --------(2)}$$

$$(1)- (2) = (xy+ xy+3x+3y+9) - (xy+3y+xy+3x)$$

$$\therefore \text{Difference}= 9$$

For any positive numbers x, y, u, v with $x > y$ and $u > v$

$$(x - y)(u - v) = xu - xv - yu + yv$$

$$(x - y)^2 = (x + y)^2 - 4xy$$

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(1) The perimeter of a rectangle is 40 centimetres and its area is 70 square centimetres. Find the area of the rectangle with each side 3 centimetres shorter.

ANSWER

Perimeter$= 2\times(l+b) = 40$cm

$l+b = \dfrac{40}{2} = 20$cm

Area$= l\times b = 70$cm^2

Area of a rectangle whose each sides 3cm shorter$=(l-3)(b-3)$

$(l$-$3)(b$-$3)\ = (l\times b) - 3l - 3b + 9$

$(l$-$3)(b$-$3)\ = (l\times b) - 3(l+b) + 9$

$= 70 - 3\times20 + 9$

$= 70 - 60 + 9 = 19$

$\therefore$ Area of a rectangle whose each sides 3cm shorter$= 19$cm^2

(2) If the sides of a rectangle are decreased by one metre, its area would be 741 square metres; if increased by one metre, it would be 861 square metres.

(i) What is the area of the rectangle ?

(ii) What is its perimeter ?

(iii) What are the lengths of its sides ?

ANSWER

(i) If the length and width are decreased by $1m$ each;
(Length $=x$, Breadth $=y$)

$(x{-}1)(y{-}1) = 741$

$xy - x - y + 1 = 741$

$xy - x - y = 741 - 1$

$xy{-}x{-}y = 740$ ---------(1)

If the length and width are increased by $1m$ each

$(x{+}1)(y{+}1) = 861$

$xy + x + y + 1 = 861$

$xy + x + y = 861 - 1$

$xy + x + y = 860$ ---------(2)

Equation (1) + (2)

$xy - x - y + xy + x + y = 740 + 860$

$2xy = 1600$

$\therefore xy = \dfrac{1600}{2} = 800m^2$

(ii) From equation number (2)

$xy + x + y = 860$

$800 + x + y = 860$

$x + y = 860 - 800 = 60$ ---------(3)

$\therefore$ Perimeter $= 2 \times (x+y)$
$= 2 \times 60$
$= 120m$

(iii)

$$(x–y)^2 = (x+y)^2 – 4xy$$

$$(x–y)^2 = 60^2 – 4\times 800$$

$$(x-y)^2 = 3600 – 3200$$

$$(x-y)^2 = 400$$

$$(x-y) = \sqrt{400}$$

$$x - y = 20 \text{ ---------(4)}$$

Equation (3) + (4)

$$\begin{aligned} x+y &= 60\ + \\ \underline{x-y} &= \underline{20} \\ 2x &= 80 \\ x &= 40 \end{aligned}$$

Substitute the value $x = 40$ in equation number (3)

$$40 + y = 60$$

$$y = 60 – 40 = 20$$

∴ **Length= 40m, Breadth = 20m**

(3) When each of two numbers are increased by one, the product becomes 1271 and when each is decreased by one, the product becomes 1131.

(i) What is the product of the numbers ?

(ii) What is their sum ?

(iii) What are the numbers ?

ANSWER

(i)   Numbers x , y

If 1 is added

$(x+1) (y+1) = 1271$
$xy + x+y + 1 = 1271$
$xy + x+y = 1271-1$
$xy+x+y = 1270$ ---------(1)

If 1 is subtracted

$(x–1)(y–1) = 1131$
$xy – x – y+1 = 1131$
$xy – x – y = 1131–1$
$xy – x – y =1130$ ---------(2)

Equation (1) + (2)
$xy+x+y+xy – x – y = 1270 + 1130$

$2xy = 2400$

$xy = \dfrac{2400}{2}$
$xy = 1200$

∴ **Product of numbers** $=1200$

(ii)   **From the equation number (1)**

$xy + x+y = 1270$
$1200 + x+ y = 1270$
$x+y = 1270 – 1200$
$x+y = 70$ ---------(3)

∴ **Sum of numbers** $= 70$

(iii)

$(x–y)^2 = (x+y)^2 – 4xy$

$(x–y)^2 = 70^2 – 4×1200$

$(x–y)^2 = 4900 – 4800$

$(x–y)^2 = 100$

$x–y = \sqrt{100}$

$x–y = 10$ ---------(4)

Equation (3) + (4)

$x + y = 70 +$
$\underline{x – y = 10}$
$2x = 80$

$x = \dfrac{80}{2} = 40$

$40 + y = 70$

$y = 70 – 40 = 30$

∴ **Numbers are 40, 30**

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(4) The product of two odd numbers just after each of two odd numbers is 285 and the product of the odd numbers just before each is 165. What are the numbers?

ANSWER

Odd numbers x, y

Two odd numbers just after the odd numbers (x+2) (y+2)

$(x+2)(y+2) = 285$

$xy + 2x + 2y + 4 = 285$

$xy + 2x + 2y = 285 – 4$

$xy + 2x + 2y = 281$ ---------(1)

Two odd numbers just before the odd numbers (x–2) (y–2)

$(x–2)(y–2) = 165$

$xy – 2x – 2y + 4 = 165$

$xy – 2x – 2y = 165 – 4$

$xy – 2x – 2y = 161$ ---------(2)

Equation (1) + (2)

$xy + 2x + 2y + xy - 2x - 2y = 281 + 161$

$2xy = 442$

$xy = \dfrac{442}{2}$

$xy = 221$

$xy + 2x + 2y = 281$

$xy + 2(x+y) = 281$

$221 + 2(x+y) = 281$

$2(x+y) = 281 - 221$

$2(x+y) = 60$

$x+y = \dfrac{60}{2} = 30$ ---------(3)

$(x-y)^2 = (x+y)^2 - 4xy$

$(x-y)^2 = 30^2 - 4 \times 221$

$(x-y)^2 = 900 - 884$

$(x-y)^2 = 16$

  $x-y = 4$ ---------(4)

Equation (3) + (4)

$x+y = 30 +$

$\underline{x - y = \ 4 \qquad\qquad}$

$2x = 34$

$x = \dfrac{34}{2} = 17$

$17 + y = 30$

$y = 30 - 17 = 13$

∴ **Odd numbers are 17**, *13*

For any positive numbers x, y, u, v with $u > v$

$$(x + y)(u - v) = xu - xv + yu - yv$$

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(1) The product of two numbers is 713 and their difference is 8.

    (i) What is the product of the larger number increased by one and the smaller number decreased by one ?

    (ii) What is the product of the larger number decreased by one and the smaller number increased by one ?

ANSWER

Larger number $= x$, **Smaller number** $= y$

$x \times y = 713$

$x - y = 8$

(i) $(x+1)(y-1) = xy - x + y - 1$

$(x+1)(y-1) = xy - (x-y) - 1$

$(x+1)(y-1) = 713 - 8 - 1$

$(x+1)(y-1) = 704$

(ii)

$(x-1)(y+1) = xy + x - y - 1$

$(x-1)(y+1) = 713 + 8 - 1$

$(x-1)(y+1) = 720$

(2) The product of two numbers is 5 more than the product of the larger of the numbers increased by one and the smaller decreased by one. How much is the increase in the product if the larger is decreased by one and the smaller increased by one?

ANSWER

Larger number = x , Smaller number = y

$xy = (x+1)(y-1) + 5$

$(x+1)(y-1) = xy - 5$

$xy - x + y - 1 = xy - 5$

$-x + y - 1 = -5$

$-(x-y) - 1 = -5$

$-(x-y) = -5 + 1$

$-(x-y) = -4$

$x - y = 4$

Then we find (x–1)(y+1)

$(x-1)(y+1) = xy + x - y - 1$

$(x-1)(y+1) = xy + 4 - 1$

$(x-1)(y+1) = xy + 3$

∴ **3 is increased with the product of the numbers**

(3) The product of the larger of two numbers increased by one and the smaller decreased by one is 540. The product of the larger decreased by one and the smaller increased by one is 560.

(i) What is the product of the numbers themselves ?

(ii) What is their difference ?

(iii) What are the numbers ?

ANSWER

Larger number = x , Smaller number = y

$(x+1)(y-1) = 540$
$xy-x+y-1 = 540$
$xy-x+y = 540 + 1$
$xy-x+y = 541$ ---------(1)

$(x-1)(y+1) = 560$
$xy+x-y-1 = 560$
$xy+x-y = 560+1$
$xy+x-y = 561$ ---------(2)

Equation (1) + (2)

$xy-x+y + xy+x-y = 541+561$

$2xy = 1102$
$xy = \dfrac{1102}{2}$
$xy = 551$

(i)   ∴ Product of numbers $= xy = 551$

(ii) $xy+x-y = 561$
            $551+x-y = 561$
            $x-y = 561 - 551$
            $x-y = 10$ ---------(3)

            ∴ Difference of numbers $= x-y = 10$

(iii)

$(x-y)^2 = (x+y)^2 - 4xy$

$10^2 = (x+y)^2 - 4 \times 551$

$100 = (x+y)^2 - 2204$

$100 + 2204 = (x-y)^2$

$2304 = (x+y)^2$

$x+y = \sqrt{2304}$

$x+y = 48$ ---------(4)

Equation (3) + (4)

$x+y = 48$
$\underline{x-y \ = 10}$
$2x = 58$

$x = \dfrac{58}{2} = 29$

$y = 48 - 29 = 19$

∴ Numbers are = 29, 19

(4) If the length of a rectangle is increased by 3 metres and the breadth decreased by 2 metres, its area would decrease by 10 square metres. If the length is decreased by 2 metres and the breadth increased by 3 metres, the area would increase by 30 square metres. Calculate the length and breadth of the rectangle.

ANSWER

Length $=x$, Breadth $= y$, Area$=xy$

$(x+3)(y-2) = xy - 10$
$xy - 2x+3y-6 = xy - 10$
$-2x + 3y = -10 + 6$
$-2x + 3y = -4$ ---------(1)

$(x-2)(y+3) = xy + 30$
$xy+ 3x-2y-6 = xy+ 30$
$3x - 2y-6 = 30$
$3x - 2y = 30+6$
$3x - 2y = 36$ ---------(2)

$(1)\times3 \longrightarrow -6x+9y= -12$ ---------(3)
$(2)\times2 \longrightarrow 6x - 4y=72$ ---------(4)

Equation (3) + (4)

$-6x+9y = -12$
$\underline{6x - 4y = 72}$
$5y= 60$

$$y= \frac{60}{5} = 12$$

$6x- 4y = 72$
$6x - 4\times12=72$
$6x - 48=72$
$6x=72+48$
$6x=120$

$$x= \frac{120}{6} =20$$
$x= 20, y = 12$

∴ Length $=20m$, Breadth $= 12m$