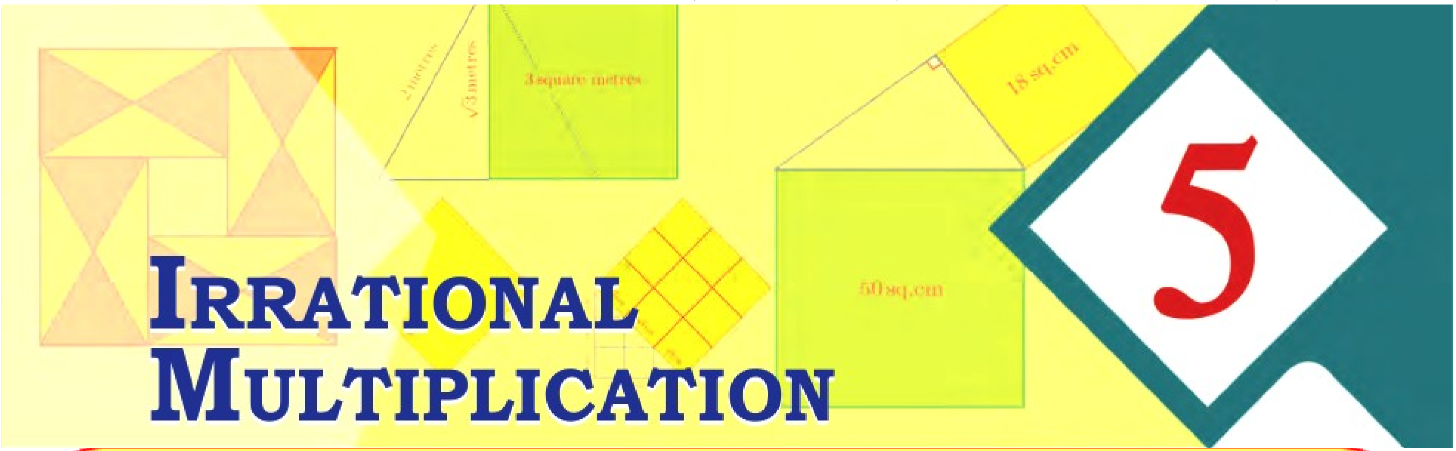
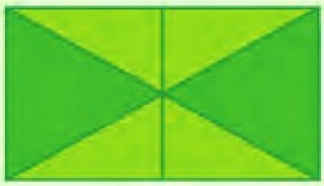


IRRATIONAL MULTIPLICATION

$\sqrt{x} \times \sqrt{y} = \sqrt{xy}$ for all positive numbers x and y

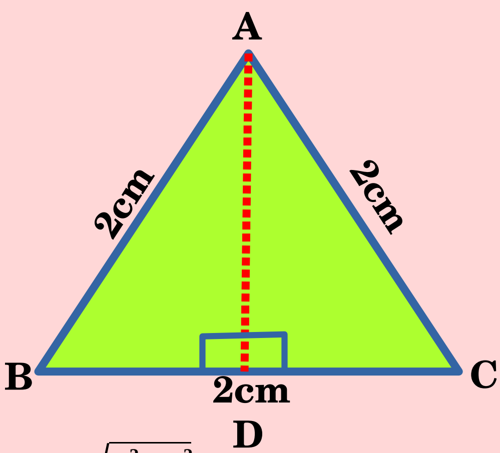
Page No: 88 & 89

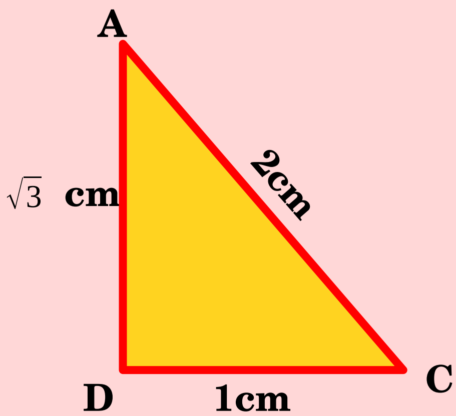
(1) We can make a rectangle using four equilateral triangles of the same size by cutting two of them along their heights and rearranging these pieces and the other two whole triangles:



If the sides of all equilateral triangles are 2 centimetres, what is the perimeter and area of the rectangle ?

Answer



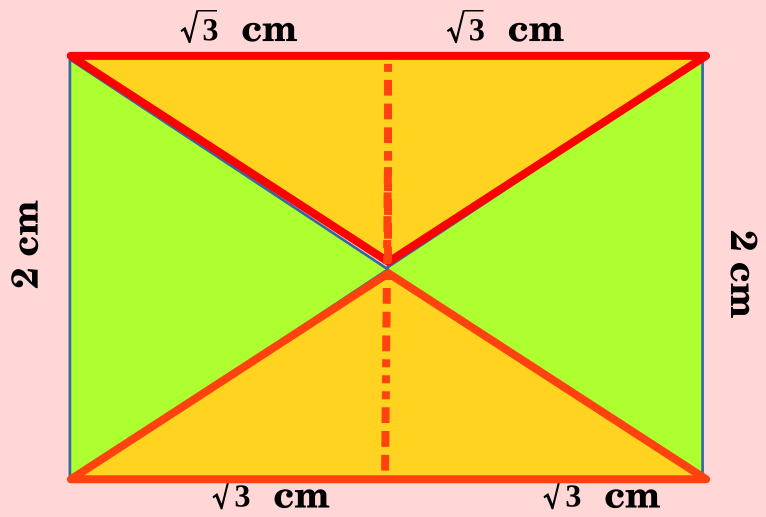


AD $= \sqrt{2^2 - 1^2}$

AD $= \sqrt{4-1} = \sqrt{3}$ **cm**

∴ Height of equilateral triangle = $\sqrt{3}$ **cm**

A rectangle using four equilateral triangles of the same size by cutting two of them along their heights and rearranging these pieces and the other two whole triangles



Length of the rectangle = $\sqrt{3}$ **cm +** $\sqrt{3}$ **m = 2** $\sqrt{3}$ **m**

Breadth of the rectangle = 2cm

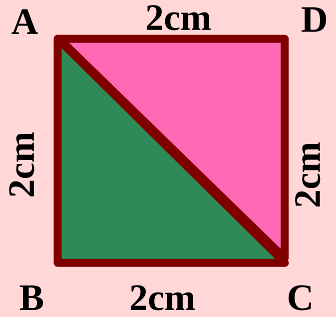
∴ Perimeter of the rectangle $= 2 + 2 + \sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3}$
$= 4 + 4\sqrt{3}$ **m**

∴ Area of the rectangle $= 2 \times 2\sqrt{3}$
$= 4\sqrt{3}$ m^2

(2) We can make a trapezium by cutting a square and an equilateral triangle with sides twice that of the square, and rearranging the pieces as below:

If the side of the square is 2 centimeters, what is the perimeter and area of the trapezium?

Answer



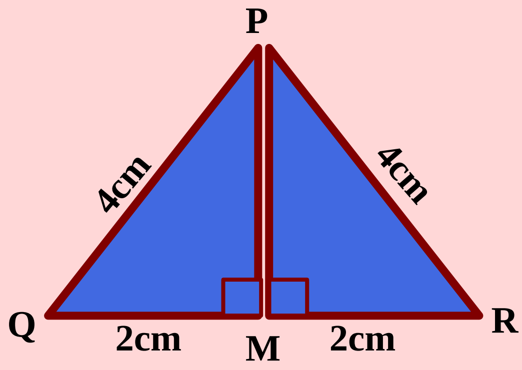
Diagonal of square ABCD , AC $= \sqrt{2^2+2^2}$

$= \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$ cm.

The length of the side of the square = 2 cm

Side length of an equilateral triangle = 2 × 2 =4 cm

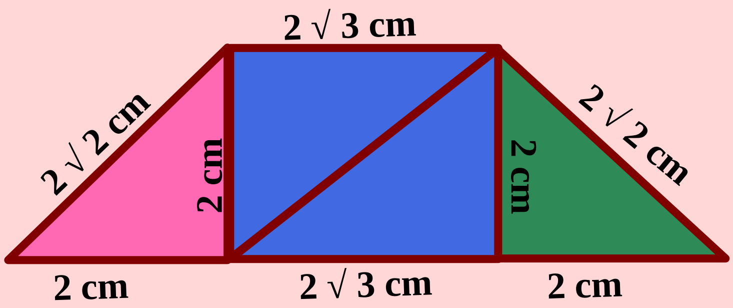


In △PQM, height PM $= \sqrt{4^2-2^2}$

$= \sqrt{16-4}$

$= \sqrt{12} = 2\sqrt{3}$ cm.

If a square and an equilateral triangle with sides twice the length of its sides form a trapezium



Perimeter of trapezium = $2 + 2 + 2\sqrt{3} + 2 + \sqrt{3}\ \ 2\sqrt{2} + 2\sqrt{2}$

$= 4\sqrt{2} + 4\sqrt{3} + 4$

Perimeter of trapezium = $4(\sqrt{2} + \sqrt{3} + 1)$ cm.

Area of trapezium = ½ × height × sum of parallel sides

Area of trapezium = ½ × h × (a + b)

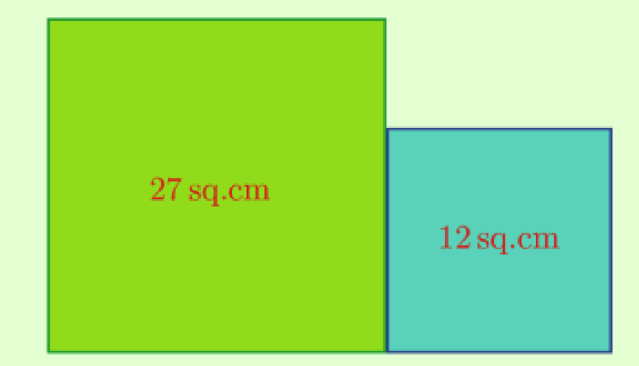
a=2√ 3cm
b= 2 + 2 + 2√ 3 = 4+4√ 3cm
h= 2cm

$= \frac{1}{2} \times 2 \times (2\sqrt{3} + 2 + 2 + 2\sqrt{3})$

$= 4 + 4\sqrt{3}$

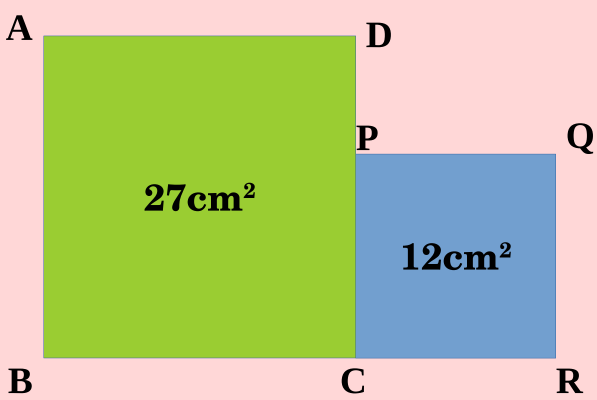
∴ Area of trapezium = $4(1 + \sqrt{3})cm^2$.

(3) The picture shows the figure formed by joining two squares:



Calculate the length of the bottom side of this figure, correct to a centimetre.

Answer



A side of a large square= $\sqrt{27} = 3\sqrt{3}$ m

A side of a small square = $\sqrt{12} = 2\sqrt{3}$ m

The length of the bottom side of this figure = BC+CR

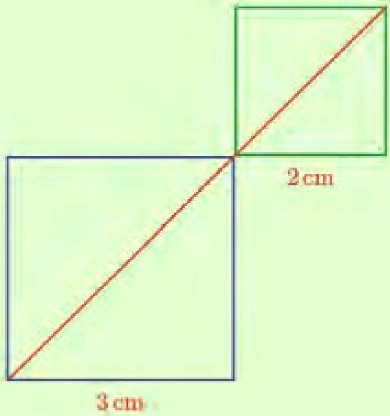
$= 3\sqrt{3} + 2\sqrt{3}$

$= 5\sqrt{3}$

$= 5 \times 1.73$

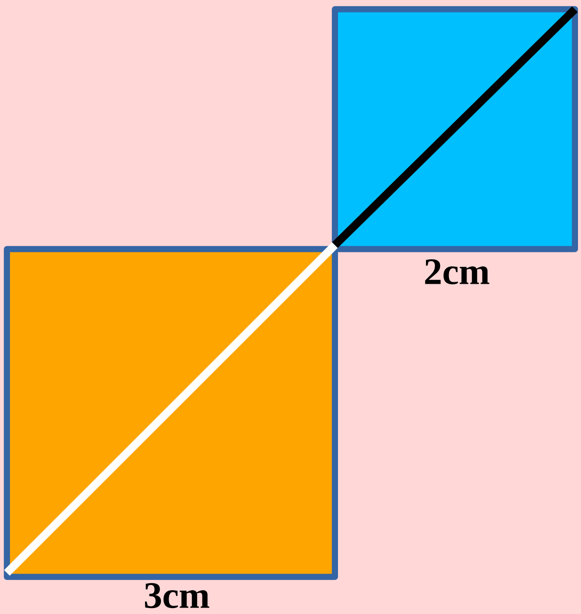
∴ The length of the bottom side of this figure = 8.65cm

(4) The figure shows two squares with two corners joined:



Find the length of the slanted line.

Answer



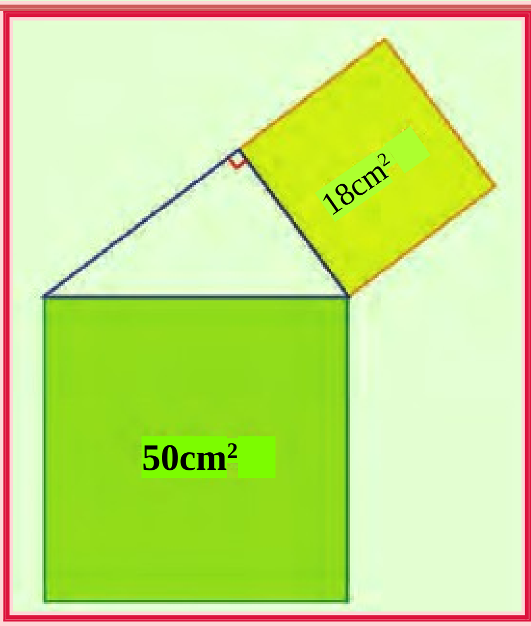
Diagonal of a square with side 'a' will be a√2

The length of the diagonal of the larger square = $3\sqrt{2}$ cm

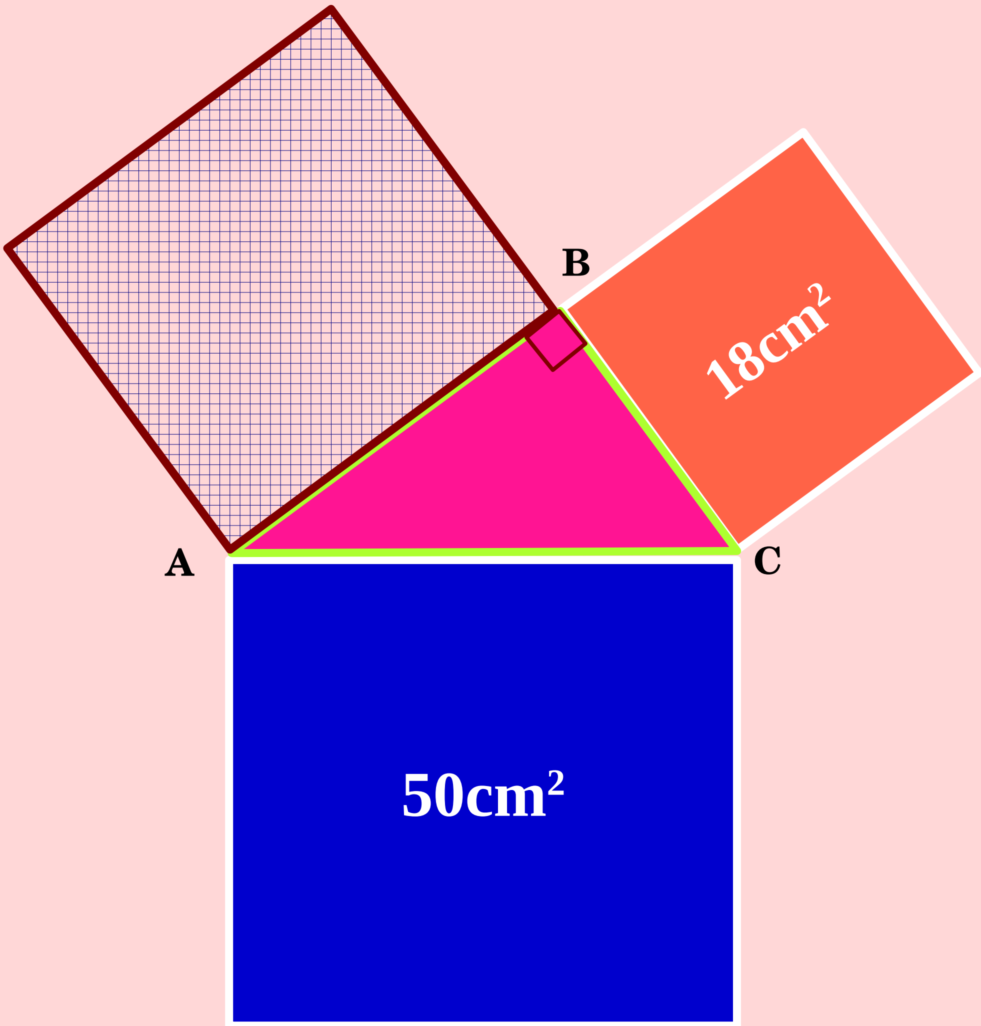
The length of the diagonal of the smaller square = $2\sqrt{2}$ m

∴ The length of the slanted line = $3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$ m

(5) Calculate the length of the third side of the right triangle in the picture and also its perimeter.



Answer



AC is from the right triangle ABC = $\sqrt{50}$

∴ side, AC= 5 $\sqrt{2}$ cm

BC is from the right triangle ABC= $\sqrt{18}$

∴ side, BC= 3 $\sqrt{2}$ cm

Area of shaded square by Pythagorean theorem = 50−18 =32cm^2

AB is from the right triangle ABC = $\sqrt{32}$

∴ Third side of right triangle, AB = 4 $\sqrt{2}$ cm

∴ Perimeter of right triangle = AC+BC+AB

= 5 $\sqrt{2}$ + 3 $\sqrt{2}$ +4 $\sqrt{2}$

= 12 $\sqrt{2}$ m.

(6) The product of some of the pairs of numbers below are natural numbers or fractions. Find those pairs.

(i) $\sqrt{3}, \sqrt{12}$ (ii) $\sqrt{3}, \sqrt{1.2}$ (iii) $\sqrt{5}, \sqrt{8}$

(iv) $\sqrt{0.5}, \sqrt{8}$ (v) $\sqrt{7\frac{1}{2}}, \sqrt{3\frac{1}{3}}$ (vi) $\sqrt{\frac{2}{5}}, \sqrt{\frac{1}{10}}$

Answer

(i) $\sqrt{3} \times \sqrt{12} = \sqrt{3\times12} = \sqrt{36}$ **= 6 natural number**

(ii) $\sqrt{3} \times \sqrt{1.2} = \sqrt{3\times1.2} = \sqrt{3.6}$ **Not a natural or a fraction**

(iii) $\sqrt{5} \times \sqrt{8} = \sqrt{5\times8} = \sqrt{40}$ **Not a natural or a fraction**

(iv) $\sqrt{0.5} \times \sqrt{8} = \sqrt{0.5\times8} = \sqrt{4.0}$ **= 2 natural number**

(v) $\sqrt{7\frac{1}{2}} \times \sqrt{3\frac{1}{3}}$

$= \sqrt{\frac{15}{2}} \times \sqrt{\frac{10}{3}} = \sqrt{\frac{15}{2}\times\frac{10}{3}} = \sqrt{\frac{50}{2}} = \sqrt{25}$ **= 5 natural number**

(vi) $\sqrt{\frac{2}{5}} \times \sqrt{\frac{1}{10}} = \sqrt{\frac{2}{5}\times\frac{1}{10}} = \sqrt{\frac{1}{25}} = \frac{1}{5}$ **fraction**

For any two positive numbers x and y

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

Page No: 91 & 92

(1) Prove that $(\sqrt{2}+1)(\sqrt{2}-1)=1$.
Using this:

(i) Compute $\frac{1}{\sqrt{2}-1}$ up to two decimal places.

(ii) Compute $\frac{1}{\sqrt{2}+1}$ up to two decimal places.

Answer

$$(\sqrt{2}+1)(\sqrt{2}-1) = (\sqrt{2})^2 - 1^2 \qquad \{(a+b)(a-b) = a^2-b^2\}$$

$$\therefore (\sqrt{2}+1)(\sqrt{2}-1) = 2-1 = 1$$

(i)

$$\frac{1}{\sqrt{2}-1} = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \frac{\sqrt{2}+1}{1} = \sqrt{2}+1$$

$$= 1.41 + 1 = 2.41$$

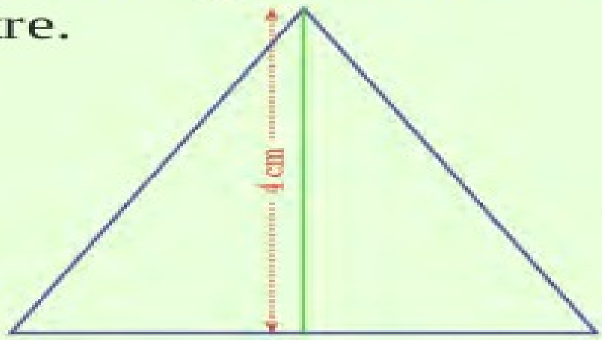
(ii)

$$\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

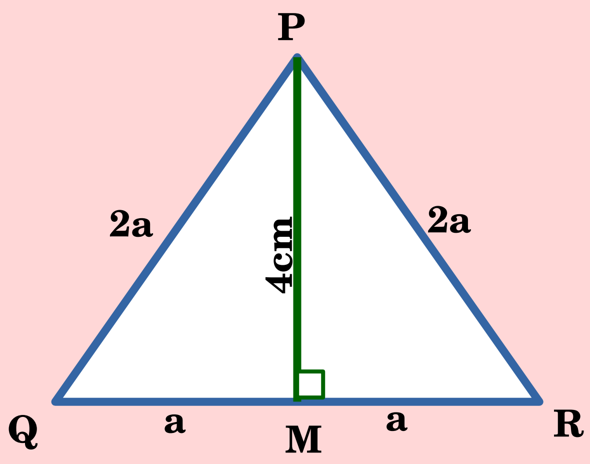
$$= \frac{\sqrt{2}-1}{1} = \sqrt{2}-1$$

$$= 1.41 - 1 = 0.41$$

(2) Compute the lengths of the sides of the equilateral triangle shown below, correct to a millimetre.



Answer



If the length of the sides is taken as 2a

From the right triangle PMQ , $PQ^2 = PM^2 + QM^2$

$$(2a)^2 = 4^2 + a^2$$

$$4a^2 - a^2 = 16$$

$$3a^2 = 16$$

$$a^2 = \frac{16}{3}$$

$$a = \sqrt{\frac{16}{3}}$$

$$a = \frac{4}{\sqrt{3}}$$

$$a = \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

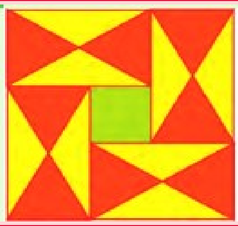
∴ A side of a triangle = $2a = 2 \times \frac{4\sqrt{3}}{3}$

$= \frac{8\sqrt{3}}{3}$

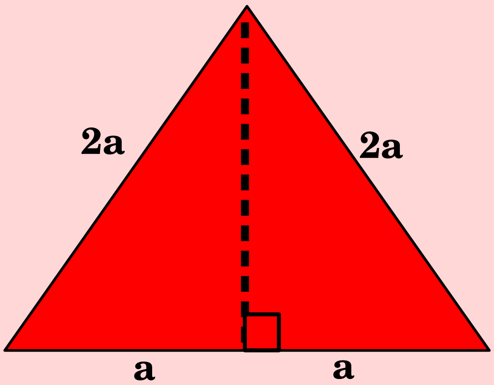
$= \frac{8 \times 1.73}{3}$

$= \frac{13.856}{3}$ = 4.618 cm

(3) All red triangles in the picture are equilateral and of the same size. What is the ratio of the sides of the outer and inner squares?

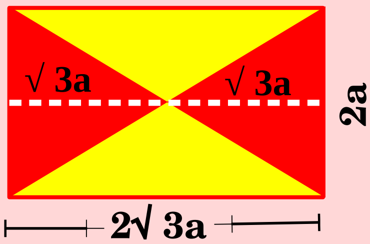


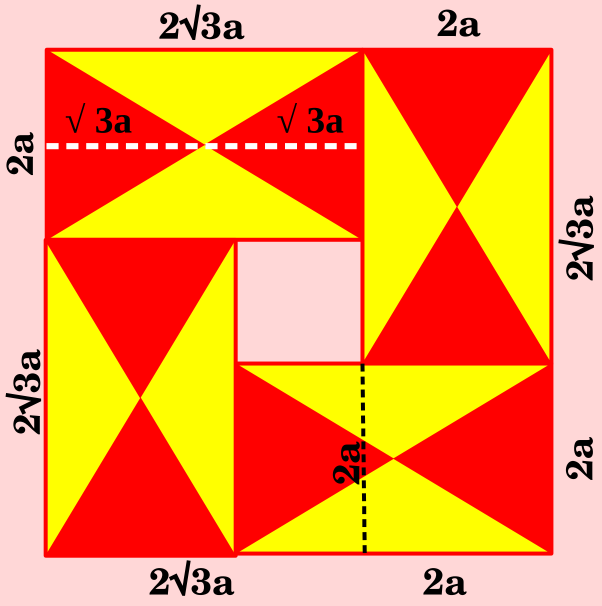
Answer



If the length of the sides is taken as 2a ,

Height of equilateral triangle = $\sqrt{(2a)^2 - a^2} = \sqrt{4a^2 - a^2} = \sqrt{3}a$





A side of outer square = $2\sqrt{3}a + 2a = 2a(\sqrt{3} + 1)$

A side of inner square = A side of outer square − (2a+ 2a)

$= 2\sqrt{3}a + 2a - (2a + 2a)$

$= 2\sqrt{3}a + 2a - 2a - 2a$

$= 2\sqrt{3}a - 2a = 2a(\sqrt{3} - 1)$

The ratio of the sides $= 2a(\sqrt{3} + 1) : 2a(\sqrt{3} - 1)$

∴ The ratio of the sides $= (\sqrt{3} + 1) : (\sqrt{3} - 1)$

(4) Prove that $\sqrt{2\frac{2}{3}} = 2\sqrt{\frac{2}{3}}$ and $\sqrt{3\frac{3}{8}} = 3\sqrt{\frac{3}{8}}$. Find other numbers like this.

Answer

$$\sqrt{2\frac{2}{3}} = \sqrt{\frac{8}{3}} = 2\sqrt{\frac{2}{3}}$$

$$\sqrt{3\frac{3}{8}} = \sqrt{\frac{27}{8}} = 3\sqrt{\frac{3}{8}}$$

If we write this in general

$$\sqrt{x\frac{x}{n}} = x\sqrt{\frac{x}{n}}$$

$$\sqrt{x+\frac{x}{n}} = \sqrt{x^2\frac{x}{n}}$$

$$\sqrt{x+\frac{x}{n}} = \sqrt{\frac{x^3}{n}}$$

$$x+\frac{x}{n} = \frac{x^3}{n}$$

$$nx + x = x^3$$

$$n+1 = x^2$$

$$n = x^2-1$$

Other numbers like this

$$\sqrt{4\frac{4}{15}} = 4\sqrt{\frac{4}{15}} \quad , \quad \sqrt{5\frac{5}{24}} = 5\sqrt{\frac{5}{24}}$$

(5) Among the pairs of numbers given below, find those for which the quotient of the first by the second is a natural number or a fraction.

(i) $\sqrt{72}, \sqrt{2}$ (ii) $\sqrt{27}, \sqrt{3}$ (iii) $\sqrt{125}, \sqrt{50}$

(iv) $\sqrt{10}, \sqrt{2}$ (v) $\sqrt{20}, \sqrt{5}$ (vi) $\sqrt{18}, \sqrt{8}$

Answer

(i) $\frac{\sqrt{72}}{\sqrt{2}} = \sqrt{\frac{72}{2}} = \sqrt{36} = 6$ **; natural number**

(ii) $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{\frac{27}{3}} = \sqrt{9} = 3$ **; natural number**

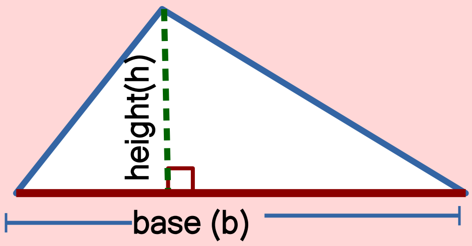
(iii) $\frac{\sqrt{125}}{\sqrt{50}} = \sqrt{\frac{125}{50}} = \sqrt{\frac{5}{2}}$ **; not a natural number or a fraction**

(iv) $\frac{\sqrt{10}}{\sqrt{2}} = \sqrt{\frac{10}{2}} = \sqrt{5}$ **; not a natural number or a fraction**

(v) $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2$ **; natural number**

(vi) $\frac{\sqrt{18}}{\sqrt{8}} = \sqrt{\frac{18}{8}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$ **; fraction**

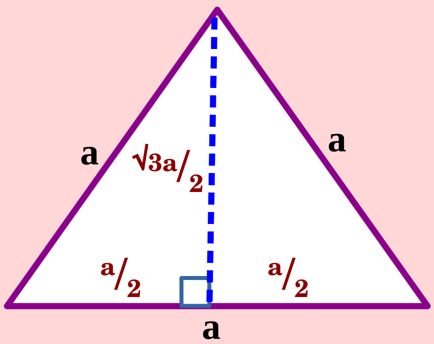
Areas of triangles



If the height is h and the base is b, then the area of the triangle = $\frac{1}{2}$ ×base ×height

∴ Area of the triangle = $\frac{1}{2} \times b \times h$

Equilateral triangle



Let the length of one side of an equilateral triangle be 'a' units. A vertical draw from vertex to base bisects the triangle divide into triangles with right angles.
This triangle will have base length $^a/_2$ and hypotenuse length 'a'.

Height of equilateral triangle , $h^2 = a^2 - (^a/_2)^2$

$h^2 = 3a^2/_4$

∴ Height of equilateral triangle, h = $^{\sqrt{3}a}/_2$

The altitude of an equilateral triangle is $\sqrt{3}$ times half the side and its area is $\sqrt{3}$ times the square of half the side.

As we know the area of triangle = $\frac{1}{2} \times b \times h$

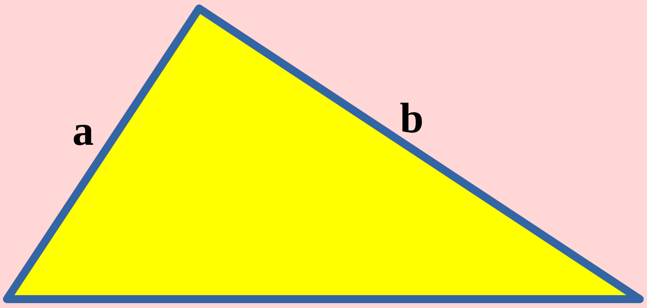
Here the area of the equilateral triangle = $\frac{1}{2} \times a \times {}^{\sqrt{3}a}/_2$

∴ Area of the equilateral triangle = $\sqrt{3}a^2/_4$

Heron's formula for triangles

To calculate the area of a triangle when it's three sides are given .

If the side lengths of a triangle are a, b and c



Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Here 's' is half of the perimeter, s = $\frac{a+b+c}{2}$

Page No: 96

(1) For each of the lengths below, calculate the area of the equilateral triangle with that as the lengths of the sides:

(i) 10 cm (ii) 5 cm (iii) $\sqrt{3}$ cm

Answer

(i) **Side = 10cm**

half of the side = 5cm

The area of an equilateral triangle is √3 times the square of half the side

$\therefore$ **Area of the equilateral triangle =** $5^2 \times \sqrt{3} = 25\sqrt{3}$ **cm²**

(ii) **Side = 5 cm**

half of the side = $\frac{5}{2}$ **cm**

$\therefore$**Area of the equilateral triangle =** $\left(\frac{5}{2}\right)^2 \times \sqrt{3}$

$= \frac{25\sqrt{3}}{4}$ **cm²**

(iii) **Side = √3cm**

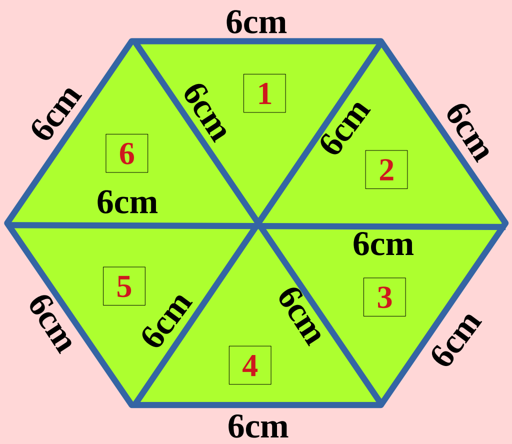
half of the side = $\frac{\sqrt{3}}{2}$ **cm**

$\therefore$ **Area of the equilateral triangle =** $\left(\frac{\sqrt{3}}{2}\right)^2 \times \sqrt{3}$

$= \frac{3\sqrt{3}}{4}$ **cm²**

(2) Calculate the area of the regular hexagon with lengths of the sides 6 centimetres.

Answer



Here, the area of regular hexagon = 6 × Area of an equilateral triangle

side of an equilateral triangle = 6 cm

half of the side = 3cm

∴ Area of the equilateral triangle = $3^2 \times \sqrt{3} = 9\sqrt{3}$ cm^2

∴ The area of regular hexagon = $6 \times 9\sqrt{3} = 54\sqrt{3}$ cm^2

(3) Calculate the perimeter and area of the equilateral triangle with height 12 centimetres.

Answer

Side of the equilateral triangle = a

Height of the equilateral triangle, h = $\frac{\sqrt{3}a}{2}$ = 12cm

One side 'a' = $\frac{12 \times 2}{\sqrt{3}} = \frac{24\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{24\sqrt{3}}{3}$

One side 'a' = $8\sqrt{3}$cm

half of the side= $4\sqrt{3}$

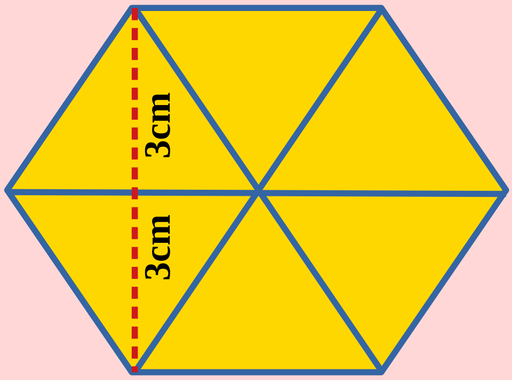
∴ Area of the equilateral triangle = $(4\sqrt{3})^2 \times \sqrt{3}$

$= 16 \times 3 \times \sqrt{3} = 48\sqrt{3}$ cm^2

Perimeter = $3 \times 8\sqrt{3} = 24\sqrt{3}$cm

(4) Calculate the perimeter and area of the regular hexagon with the distance between parallel sides 6 centimetres.

Answer



The height of an equilateral triangle whose side 'a' is , h = $\sqrt{3}a/2$ = 3cm

One side 'a' = $\frac{3 \times 2}{\sqrt{3}}$

$= \frac{6\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$= \frac{6\sqrt{3}}{3}$ = $2\sqrt{3}$cm

Perimeter of regular hexagon = 6 × 2√3 = 12√3cm

Area of regular hexagon = 6 × Area of an equilateral triangle

Side of the equilateral triangle = 2√3cm

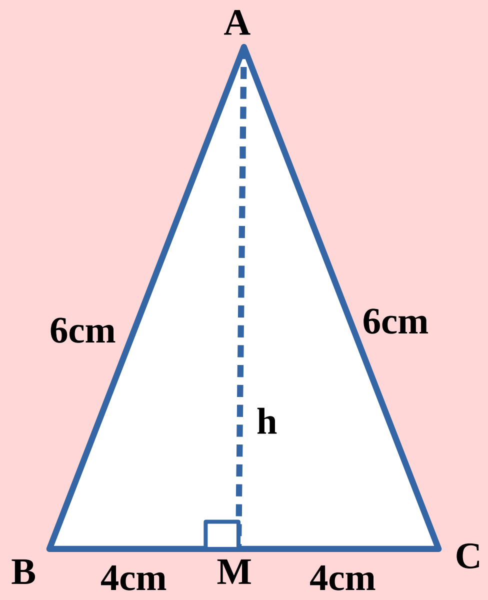
half of the side = √3cm

∴Area of the equilateral triangle= $(\sqrt{3})^2 \times \sqrt{3} = 3\sqrt{3}$ cm^2

∴The area of regular hexagon = 6 × 3√3 = 18√3cm^2

(5) Calculate the height and area of the triangle with sides 8 centimetres, 6 centimetres, 6 centimetres.

Answer



In $\triangle AMB$, $h^2 = AB^2 - BM^2$

$h^2 = 6^2 - 4^2$

$h^2 = 36 - 16$

$h^2 = 20$

$\therefore$ Height , $h = \sqrt{20}$ cm $= 2\sqrt{5}$cm

Area of $\triangle ABC = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 8 \times 2\sqrt{5}$

$\therefore$ Area of $\triangle ABC = 8\sqrt{5}cm^2$

(6) For each of the set of three lengths given below, calculate the area of the triangle with these as the lengths of sides:

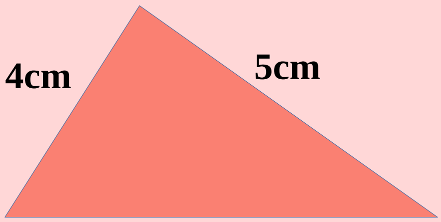
(i) 4 cm, 5 cm, 7cm

(ii) 4cm, 13 cm, 15 cm

(iii) 5 cm, 12 cm, 13 cm

Answer

(i) 4cm, 5cm, 7cm



a = 4cm, b = 5cm, c = 7cm

$s = \frac{a+b+c}{2} = \frac{4+5+7}{2} = \frac{16}{2} = 8\text{cm}$

s−a = 8−4 = 4cm

s−b = 8−5 = 3cm

s−c = 8−7 = 1cm

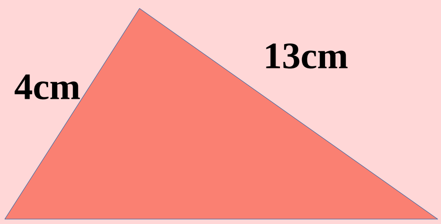
Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{8 \times 4 \times 3 \times 1}$

$= \sqrt{96}$

∴ **Area of triangle =** $4\sqrt{6}\text{cm}^2$

ii) 4cm, 13cm, 15cm



a = 4cm, b = 13cm, c = 15cm

s = $\frac{a+b+c}{2}$ $\frac{4+13+15}{2}$ **=** $\frac{32}{2}$ **= 16cm**

s−a= 16−4 = 12cm

s−b= 16−13 = 3cm

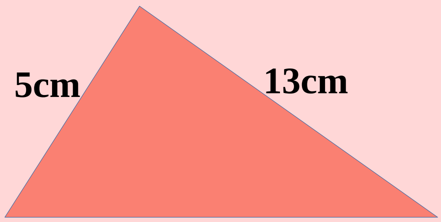
s−c= 16−15 = 1cm

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Area of triangle= $\sqrt{16\times12\times3\times1}$

$\therefore$ **Area of triangle=** $\sqrt{576}$ **= 24cm^2**

(iii) 5cm, 12cm, 13cm



a = 5cm, b = 12cm, c = 13cm

$$s = \frac{a+b+c}{2} \quad \frac{5+12+13}{2} = \frac{30}{2} = 15\text{cm}$$

s−a = 15−5 = 10cm

s−b = 15−12 = 3cm

s−c = 15−13 = 2cm

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Area of triangle = $\sqrt{15\times 10\times 3\times 2}$

$\therefore$ **Area of triangle =** $\sqrt{900}$ **= 30cm^2**