1 ARITHMETIC SEQUENCES

Number sequence.

A collection of numbers, which are ordered as the first, second, third and so on like this, according to some rule, is called a number sequence.

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(1) We can make triangles by stacking dots:

Write the number of dots in each triangle. Calculate the number of dots needed to make the next three triangles in this pattern.

Answer

The number of dots in each triangles are 3, 6, 10

The number of dots needed to make the next three triangles will be:

10 + 5 = 15

15 + 6 = 21

21 + 7 = 28

The number of dots needed to make the next three triangles are 15, 21, 28 dots.

(2) From the sequence equilateral triangle, square, regular pentagon and so on of regular polygons, form the following sequences

Number of sides	3, 4, 5, 6, ...
Sum of inner angles	
Sum of outer angles	
An inner angle	
An outer angle	

Answer

(i). No. of triangles of a regular polygon having no. of sides are 3, 4, 5 ... n respectively is given as 1, 2, 3, 4.
Sum of interior angles of a triangle is 180°.
Then the sequence of Sum of interior angles
= 180, 180 × 2, 180 × 3 .... = 180, 360, 540, ...

(ii). Sum of exterior angles of any polygons having any number of sides is always 360°.
Sum of exterior angles = 360, 360, 360, ...

(iii). One inner angle = $\frac{180}{3}$, $\frac{360}{4}$, $\frac{540}{5}$, ... = 60, 90, 108, ...

(iv). One outer angle = $\frac{360}{3}$, $\frac{360}{4}$, $\frac{360}{5}$, $\frac{360}{6}$ .... = 120, 90, 72, ...

(3) Write the sequence of natural numbers which leave remainder 1 on division by 3, and the sequence of natural numbers which leave remainder 2 on division by 3.

Answer

(i) The numbers that leave 1 as remainder when divided by 3 are 1,4,7,10,13,...
(We can write these numbers (3×0)+1, (3×1)+1, (3×2)+1, ...)

(ii) The numbers that leave 2 as remainder when divided by 3 are 2,5,8,11,14, ...
(We can write these numbers (3×0)+2, (3×1)+2, (3×2)+2, ...)

(4) Write in ascending order, the sequence of natural numbers with last digit 1 or 6. Describe this sequence in two other ways.

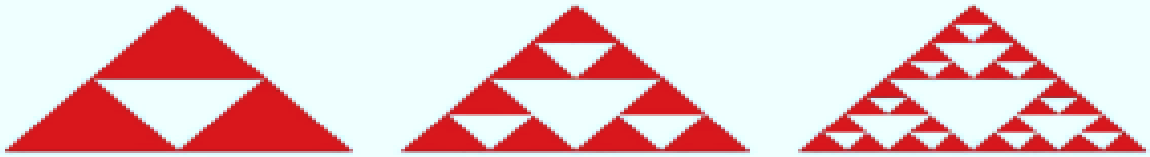
Answer

1, 6, 11, 16, 21, ...

Numbers, each with difference of 5 and starting from 1.

Numbers, when divided by 5, leaves 1 as remainder.

(5) See these figures:



The first picture shows an equilateral triangle with the smaller triangle got by joining the midpoints of sides cut off. The second picture shows the same thing done on each of the three triangles in the first picture. The third picture shows the same thing done on the second picture.

(i) How many red triangles are there in each picture?

(ii) Taking the area of whole uncut triangle as 1, compute the area of a small triangle in each picture.

(iii) What is the total area of all the red triangles in each picture?

(iv) Write the first five terms of each of the three sequences got by continuing this process.

Answer

i. **The red triangles in the first picture = 3**

The red triangles in the second picture = 9

The red triangles in the third picture = 27

ii. Area of the original uncut triangle = 1

Area of the first triangle = $\frac{1}{4}$

Area of red triangles in the second picture = $\frac{1}{16}$

Area of red triangles in the third picture = $\frac{1}{64}$

iii. Total area of red triangle in the first picture = = $3\times \frac{1}{4} = \frac{3}{4}$

Total area of red triangles in the second picture = $9\times \frac{1}{16} = \frac{9}{16}$

Total area of red triangles in the third picture = $27\times \frac{1}{64} = \frac{27}{64}$

iv. (a). First sequence is $3,9,27,....3^n$

$\therefore$ First five terms are 3,9,27,81,243

(b). Second sequence is $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, ... \frac{1}{4^n}$

$\therefore$ First five terms are $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}, \frac{1}{1024}$

(c). Third sequence is $3\times \frac{1}{4}, 9\times \frac{1}{16}, 27\times \frac{1}{64}, ... \left(\frac{3}{4}\right)^n$

$= \frac{3}{4}, \frac{9}{16}, \frac{27}{64}, ..., \left(\frac{3}{4}\right)^n$

$\therefore$ First five terms are $\frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \frac{81}{256}, \frac{243}{1024}$

Arithmetic sequences

A sequence starting with a number and proceeding by adding one number again and again, is called an arithmetic sequence.

*** The numbers in an arithmetic sequence are called the terms of the sequence.**

For example: 10,13,16,19, 22, ... is an arithmetic sequence.

1st term	2nd term	3rd term	4th term	5th term	.....	.....
10	13	16	19	22	....	.....

*** The number, got by subtracting the previous term from any term, is called the common difference(d) of the arithmetic sequence.**

For example: 10,13,16,19, 22, ... is an arithmetic sequence.

Common difference(d) = 13 -10 = 16 -13 = 19 -16 = 22 -19= 3

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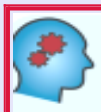


(1) Check whether each of the sequences given below are arithmetic sequences. Give reasons also. Find the common differences of the arithmetic sequences:

(i) Natural numbers leaving remainder 1 on division by 4

(ii) Natural numbers leaving remainder 1 or 2 on division by 4

(iii) Squares of natural numbers

(iv) Reciprocals of natural numbers

(v) Powers of 2

(vi) Half of the odd numbers

Answer

(i)

The sequence is 1, 5, 9, 13, ...

{(4×0)+1, (4×1)+1, (4×2)+1, ...}

Common difference(d) = 5-1 = 9-5 = 13-9 = 4

Here the common difference is same.

∴ It is an arithmetic sequence.

(ii)

The sequence is 1, 2, 5, 6, 9, 10, ...

The differences between consecutive terms are not constant (2 - 1 = 1, 5 - 2 = 3, etc.). Thus, it is not an arithmetic sequence.

(iii)

The sequence is: 1^2, 2^2 , 3^2, 4^2, ...

The sequence is: 1, 4, 9, 16, 25, ...

The differences between consecutive terms are not constant (4 - 1 = 3, 9 - 4 = 5, etc.). Thus, it is not an arithmetic sequence.

(iv)

The sequence is: 1, $\frac{1}{2}$ $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, ...

The differences between consecutive terms are not constant ($\frac{1}{2}$ −1= $\frac{-1}{2}$, $\frac{1}{3}$ − $\frac{1}{2}$ = $\frac{-1}{6}$, etc.).

Thus, it is not an arithmetic sequence.

(v)

The sequence is: $2^0, 2^1, 2^2, 2^3, ...$

The sequence is: 1, 2, 4, 8, 16, ... **(remember $2^0=1$)**

The differences between consecutive terms are not constant (2 – 1 = 1, 4 – 2 = 2, etc.). Thus, it is not an arithmetic sequence.

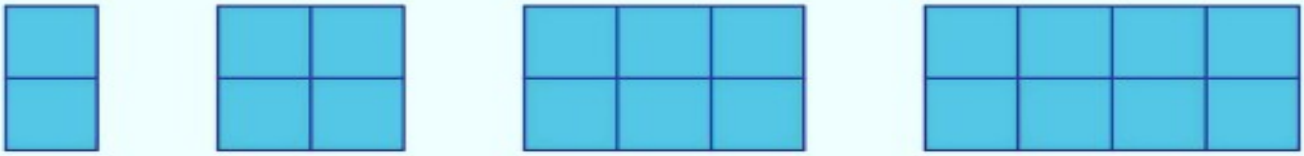
(vi)

The sequence is: $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$

The sequence is: 0.5, 1.5, 2.5, 3.5, 4.5, ...

The difference between consecutive terms is constant (1.5 – 0.5 = 1, 2.5 – 1.5 =1, etc.).Thus, it is an arithmetic sequence.

(2) See these pictures:



(i) How many small squares are there in each picture?

(ii) How many large squares?

(iii) How many squares in all in each picture?

If we continue the pattern of pictures, are the sequences above arithmetic sequences?

Answer

(i)

Number of small squares in the first picture= 2

Number of small squares in the second picture= 4

Number of small squares in the third picture= 6

Number of small squares in the fourth picture= 8

The sequence is: 2,4,6,8,.... is an arithmetic sequence with common difference 2 (4–2=2, 6–4=2, 8–6=2, etc)

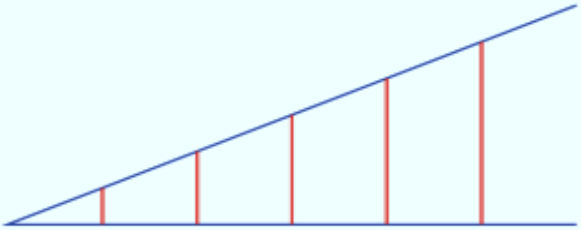
(ii)

Number of large squares in first picture= 0
Number of large squares in second picture= 1
Number of large squares in third picture= 2
Number of large squares in fourth picture= 3
The sequence is: 0,1,2,3,.... is an arithmetic sequence with common difference 1 (1–0=1, 2–1=1, 3–2=1, etc)

(iii)

Total squares in the first picture= 2
Total squares in the second picture= 5
Total squares in the third picture= 8
Total squares in the fourth picture= 11
The sequence is: 2,5,8,11,.... is an arithmetic sequence with common difference 3 (5–2=3, 8–5=3, 11–8=3, etc)

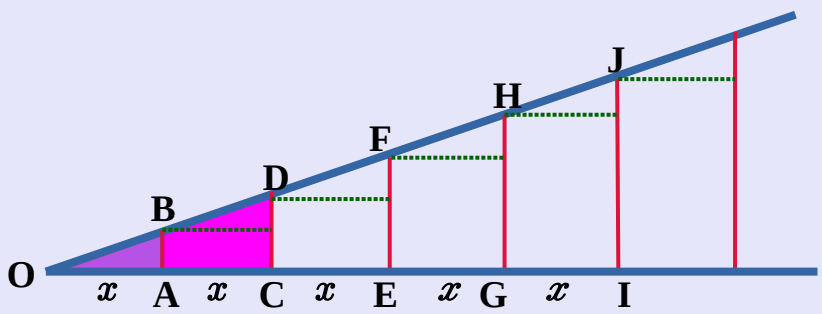
(3) In the picture below, the perpendiculars drawn from the bottom line are equally spaced.



Show that the sequence of the heights of the perpendiculars, on continuing this, form an arithmetic sequence.

(Hint: Draw perpendiculars from the top of each perpendicular to the next perpendicular)

Answer



OA=AC=CE=EG=GI= x

ΔOAB , ΔOCD are similar right angled triangle .

(∠O =∠ COD,∠OAB=∠OCD=90°)

(*If the corresponding angles of two triangles are equal, their corresponding sides will maintain the same ratio, so the triangles are similar.)*

∴ Triangles are similar.

$$\frac{OA}{OC} = \frac{AB}{CD} = \frac{x}{2x}$$

CD=2AB

similarly EF=3AB, sequence of length of perpendicular AB, 2AB, 3AB, 4AB,.....

That is AB is the common difference , so perpendicular lengths are in arithmetic sequence

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Position and term

Common difference(d)= $\frac{\textit{Term difference}}{\textit{position difference}}$

In each of the arithmetic sequences below, some of the terms are not written, but indicated by ◯. Find out these numbers:

(i) 24, 42, ◯, ◯, ... (ii) ◯, 24, 42, ◯, ...

(iii) ◯, ◯, 24, 42, ... (iv) 24, ◯, 42, ◯, ...

(v) ◯, 24, ◯, 42, ... (vi) 24, ◯, ◯, 42, ...

Answer

(i).

Common difference (d)= 42−24=18

Third term= 42+18=60

Fourth term= 60+18=78

(ii).

Common difference (d)= 42−24=18

First term= 24−18=6

Fourth term= 42+18=60

(iii).

Common difference (d)= 42−24=18

Second term= 24−18=6

First term = 6−18= −12

(iv).

First term + 2 × Common difference = Third term

24 + (3−1) × Common difference = 42

Common difference= $\frac{42-24}{2} = \frac{18}{2} = 9$

Second term= 24+9=33

Fourth term= 42+9=51

(v).

Common difference= $\frac{42-24}{4-2} = \frac{18}{2} = 9$

First term = 24–9=15

Third term=24+9=33

(vi).

Common difference= $\frac{42-24}{4-1} = \frac{18}{3} = 6$

Second term= 24+6=30

Third term=30+6=36

The two terms in specific positions of some arithmetic sequences are given below. Write the first five terms of each:

(i) 3^{rd} term 34 (ii) 3^{rd} term 43 (iii) 3^{rd} term 2 (iv) 5^{th} term 8 (v) 5^{th} term 7

6^{th} term 67 6^{th} term 76 5^{th} term 3 9^{th} term 10 7^{th} term 5

Answer

(i).

Common difference= $\frac{67-34}{6-3} = \frac{33}{3} = 11$

Second term= 34–11=23

First term =23–11=12

Fourth term= 34+11=45

Fifth term=45 +11=56

The first five terms of the sequence is 12,23,34,45,56

OR

First term = Third term– (3^{rd}position - 1^{st}position)×Common difference

= 34–2×11= 34–22=12

Second term=Third term- (3^{rd}position – 2^{nd}position)×Common difference

=34–1×11=34–11=23

If we take 6^{th} and 4^{th} terms

Fourth term= 6^{th} term–(6^{th}position – 4^{th}position)×Common difference

= 67– 2×11= 67–22=45

If we take 6^{th} and 5^{th} terms

Fifth term= 6^{th} term–(6^{th}position – 5^{th}position)×Common difference

= 67– 1×11= 67–11=56

(ii).

Common difference= $\frac{76-43}{6-3} = \frac{33}{3} = 11$

First term =43–2 ×11=43–22=21

Second term= 21+11=32

Fourth term= 43+11=54

Fifth term=54+11=65

The first five terms of the sequence is 21,32,43,54,65

(iii).

Common difference= $\frac{3-2}{5-3} = \frac{1}{2}$

First term = 2– 2 × $\frac{1}{2}$ = 2–1 = 1

Second term= 1+ $\frac{1}{2}$ = 1 $\frac{1}{2}$

Fourth term= 2+ $\frac{1}{2}$ = 2 $\frac{1}{2}$

The first five terms of the sequence is 1, 1 $\frac{1}{2}$, 2, 2 $\frac{1}{2}$, 3

(iv).

Common difference= $\frac{10-8}{9-5} = \frac{2}{4} = \frac{1}{2}$

First term = 8 – (5–1) × $\frac{1}{2}$

= 8– 4× $\frac{1}{2}$ =8–2=6

Second term= 6+ $\frac{1}{2}$ = 6 $\frac{1}{2}$

Third term = 6 $\frac{1}{2}$ + $\frac{1}{2}$ = 7

Fourth term=7+ $\frac{1}{2}$ = 7 $\frac{1}{2}$

The first five terms of the sequence is 6,6 $\frac{1}{2}$,7, 7 $\frac{1}{2}$, 8.

(v).

Common difference= $\frac{5-7}{7-5} = \frac{-2}{2} = -1$

First term = 7–(5–1) ×–1

= 7–4×–1

= 7+4=11

Second term= 11+(–1) = 10

Third term = 10+(–1)= 9

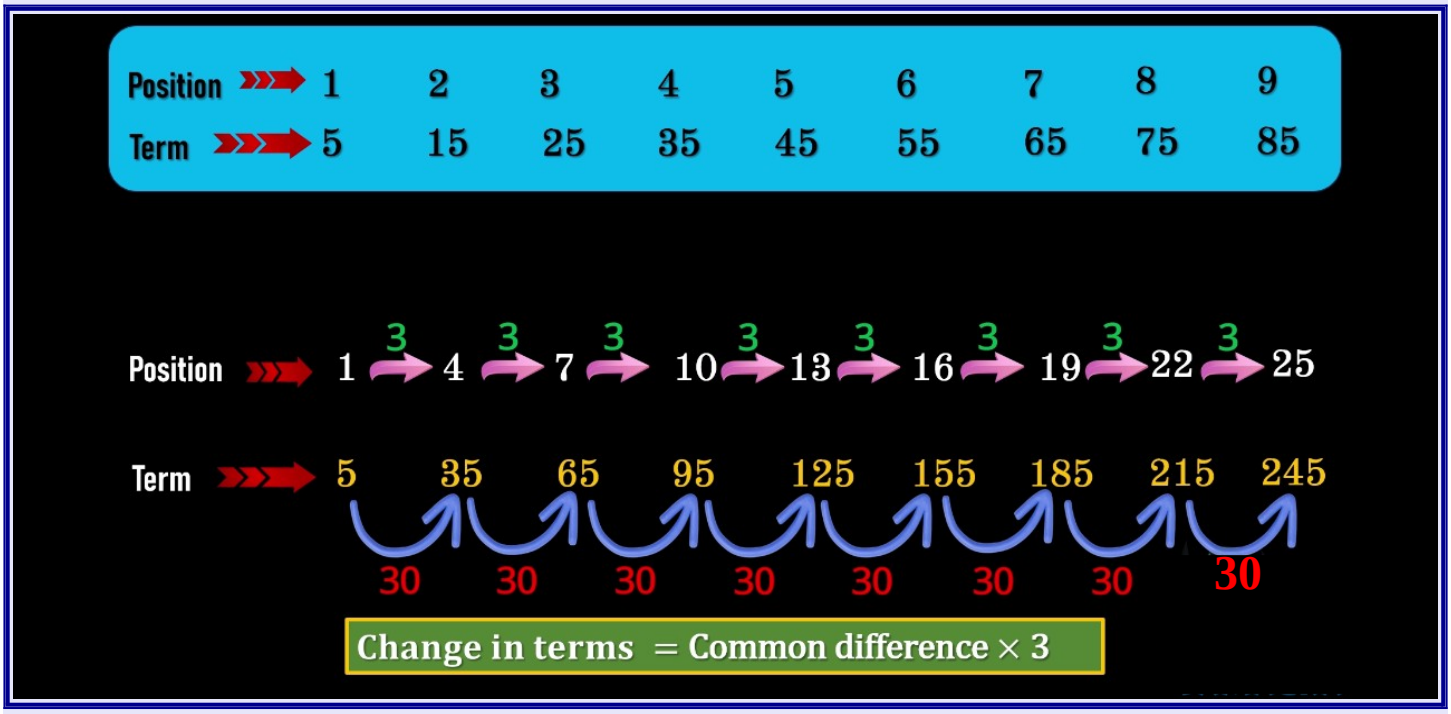
Fourth term= 9+(–1)= 8

The first five terms of the sequence is 11,10,9,8,7.

Changes in position and terms

➔ ***In any arithmetic sequence, the change in terms is the product of the change in position and a fixed number.***

➔ ***In any arithmetic sequence, the change in terms is proportional to the change in position.***



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(1) What is the 25th term of the arithmetic sequence 1, 11, 21, ... ?

Answer

Change in terms 1st and 2nd = 11–1=10
Change in positions 1 and 25 = 25–1=24
25th term = 1st term + {(25–1) × 10}
= 1+ 24 × 10
= 1 + 240 = 241

(If we take 2nd term and 25th term
25th term= 2nd term + {(25–2) × 10}
= 11+23×10
= 11+ 230=241)

(2) The 10^{th} term of an arithmetic sequence is 46 and its 11^{th} term is 51

(i) What is its first term?

(ii) Write the first five terms of the sequence

Answer

i) Change in terms 10^{th} and 11^{th} = 51–46=5
Change in positions 1 and 10 = 10–1=9
First term = 46– {(10–1) × 5}
= 46– 9 × 5
= 46–45 = 1

ii) First 5 terms of the sequence = 1, 6, 11, 16, 21.

(3) What is the 21^{st} term of the arithmetic sequence 100, 95, 90, ...?

Answer

Change in terms 1^{st} and 2^{nd} = 100–95=5
Change in positions 1 and 21 = 21–1=20
21^{st} term = 100–(20 × 5)
= 100–100= 0

(4) The 10^{th} term of an arithmetic sequence is 56 and its 11^{th} term is 51

(i) What is its first term ?

(ii) Write the first five terms of the sequence

Answer

i) Change in terms 10^{th} and 11^{th} = 56–51=5
Change in positions 1 and 10 = 10–1=9
First term = 56 + (9 × 5)
= 56 + 45 = 101

ii) First 5 terms of the sequence = 101, 96, 91, 86, 81

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(1) The 3rd term of an arithmetic sequence is 15 and the 8th term is 35

(i) What is its 13th term?

(ii) What is its 23rd term?

Answer

i) Change in terms 3^{rd} and 8^{th} = 35−15=20

Change in positions 3 and 8 = 8−3=5

Change in positions 8 and 13 = 13−8=5

When the position increase by 5 , the term increase by $5 \times \frac{20}{5}$

$= 5 \times 4 = 20$

13^{th} term = 35 +20= 55

ii) Change in positions 8^{th} and 23^{rd} = 23−8=15

When the position increase by 15 , the term increase by $15 \times \frac{20}{5}$

$= 15 \times 4 = 60$

23^{rd} term = 35 +60= 95

(2) The 5th term of an arithmetic sequence is 21 and the 9th term is 41

(i) What is its first term?

(ii) What is its 3rd term?

Answer

i) Change in terms 5^{th} and 9^{th} = 41−21=20

Change in positions 5 and 9 = 9−5=4

Change in positions 1 and 5 = 5−1=4

When the position increase by 4 , the term decrease by $4 \times \frac{20}{4}$

$= 4 \times 5 = 20$

First term = 21 − 20= 1

ii) Change in positions 3^{rd} and 5^{th} = 5−3=2

When the position increase by 2 , the term decrease by $2 \times \frac{20}{4}$

$= 2 \times 5 = 10$

3^{rd} term = 21 − 10= 11

(3) The 4^{th} term of an arithmetic sequence is 61 and the 7^{th} term is 31

(i) What is its 10^{th} term?

(ii) What is its first term?

Answer

i) **Change in terms 4^{th} and 7^{th} = 61−31=30**

Change in positions 4 and 7 = 7−4=3

Change in positions 7 and 10 = 10−7=3

When the position increase by 3 , the term decrease by $3 \times \frac{30}{3}$

$= 3 \times 10 = 30$

10^{th} term = 31 − 30= 1

ii) **Change in positions 1^{st} and 7^{th} = 7−1=6**

When the position increase by 6 , the term increase by $6 \times \frac{30}{3}$

$= 6 \times 10 = 60$

First term = 31 + 60= 91

(4) The 5^{th} term of an arithmetic sequence is 10 and the 10^{th} term is 5

(i) What is its 15^{th} term?

(ii) What is its 25^{th} term?

Answer

i) **Change in terms 5^{th} and 10^{th} = 10−5=5**

Change in positions 5 and 10 = 10−5=5

Change in positions 10 and 15 = 15−10=5

When the position increase by 5 , the term decrease by $5 \times \frac{5}{5}$

$= 5 \times 1 = 5$

15^{th} term =5−5= 0

ii) **Change in positions 10 and 25 = 25−10=15**

When the position increase by 15 ,the term decrease by $15 \times \frac{5}{5}$

$= 15 \times 1 = 15$

25^{th} term = 5 − 15= −10

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(1) Is 101 a term of the arithmetic sequence 13, 24, 35, ...? What about 1001?

Answer

Common difference = 35−24=11

101−13=88

88÷11=8

The difference between terms is a multiple of the common difference. So 101 is the 9^{th} term of this sequence .

(2) In the table below, some arithmetic sequences are given and two numbers against each. Check whether the numbers are terms of the respective sequences:

Sequence	Numbers	Yes/No
11, 22, 33, ...	123	
	132	
12, 23, 34, ...	100	
	1000	
21, 32, 43, ...	100	
	1000	
$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$	3	
	4	
$\frac{3}{4}, 1\frac{1}{2}, 2\frac{1}{4}, \ldots$	3	
	4	

Answer

i) **11,22,33,.....**

Common difference =22−11=11

123−11=112 is not a multiple of 11

∴ 123 is not a term of this sequence

132−11=121

121÷11=11

The difference between terms is a multiple of the common difference. So 132 is a term of this sequence.

ii) **12,23,34,.....**

Common difference = 23−12=11

100−12=88

88÷11=8

So 100 is a term of this sequence.

1000−12=988 is not a multiple of 11

∴ 1000 is not a term of this sequence.

iii) **21,32,43,.....**

Common difference = 32−21=11

100−21=79 is not a multiple of 11

∴ 100 is not a term of this sequence.

1000−21= 979

979÷11=89

So 1000 is a term of this sequence.

iv) $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, ...

Common difference = $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$3 - \frac{1}{4} = 2\frac{3}{4}$

$2\frac{3}{4} \div \frac{1}{4}$

$\frac{11}{4} \div \frac{1}{4}$

$\frac{11}{4} \times \frac{4}{1} = 11$

So 3 is a term of this sequence.

$4 - \frac{1}{4} = 3\frac{3}{4}$

$= 3\frac{3}{4} \div \frac{1}{4}$

$= \frac{15}{4} \div \frac{1}{4}$

$= \frac{15}{4} \times \frac{4}{1} = 15$

So 4 is a term of this sequence.

v) $\frac{3}{4}$, $1\ \frac{1}{2}$, $2\ \frac{1}{4}$, ...

Common difference = $1\ \frac{1}{2} - \frac{3}{4} = \frac{3}{4}$

$3 - \frac{3}{4} = 2\ \frac{1}{4}$

$2\ \frac{1}{4} \div \frac{3}{4}$

$\frac{9}{4} \div \frac{3}{4}$

$\frac{9}{4} \times \frac{4}{3} = 3$

So 3 is a term of this sequence.

$4 - \frac{3}{4} = 3\ \frac{1}{4}$

$3\ \frac{1}{4} \div \frac{3}{4}$

$\frac{13}{4} \div \frac{3}{4}$

$\frac{13}{4} \times \frac{4}{3} = \frac{13}{3} = 4\ \frac{1}{3}$

So 4 is not a term of this sequence.

(3) In the table above, find the position of the numbers that are terms of the respective sequences.

Answer

i) **11,22,33,.....**

132 is the 12^{th} term of this sequence.

ii) **12, 23, 34, ...**

100 is the 9^{th} term of this sequence.

iii) **21, 32, 43, ...**

1000 is the 90^{th} term of this sequence.

iv) $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, ...

3 is the 12^{th} term of this sequence.

4 is the 16^{th} term of this sequence.

v) $\frac{3}{4}$, $1\ \frac{1}{2}$, $2\ \frac{1}{4}$, ...

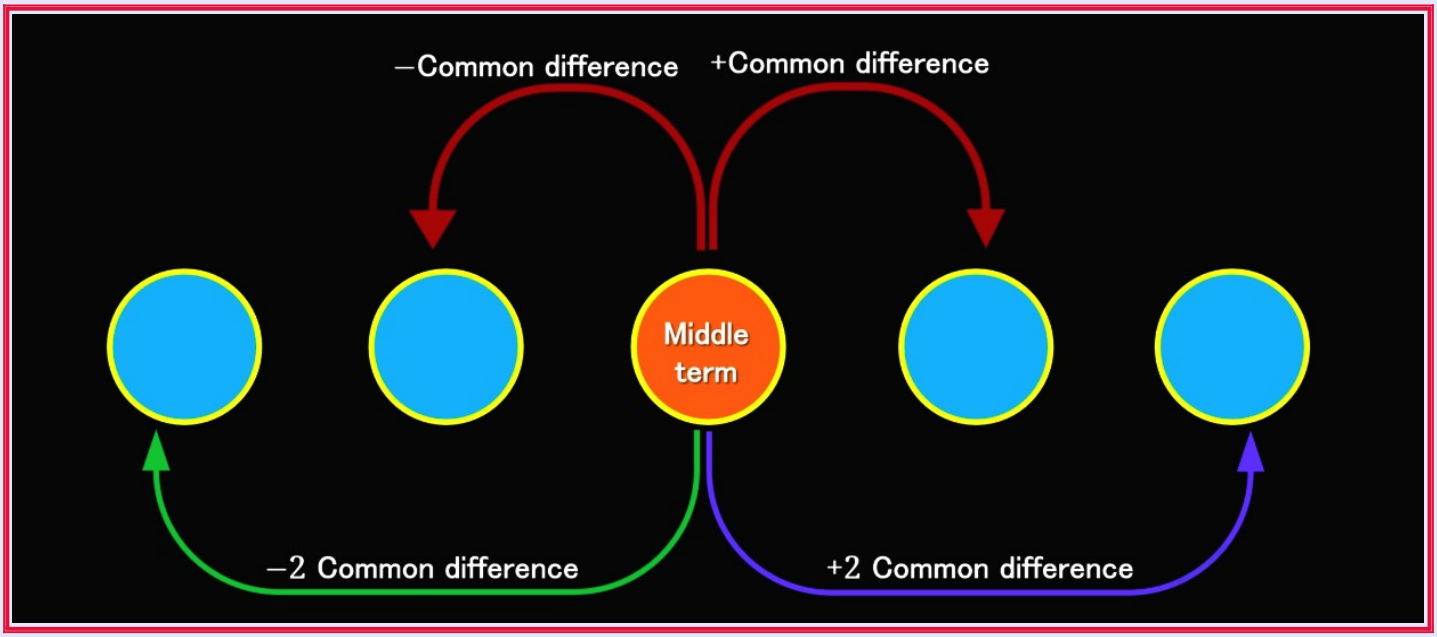
3 is the 4^{th} term of this sequence.

Term connections

(i) In an arithmetic sequence, the sum of the two terms, at the same distance behind and ahead a term, is twice this term

(ii) In an arithmetic sequence, the sum of a term and the consecutive terms at the same distance behind and ahead, is the product of this term and the number of terms

The sum of an odd number of consecutive terms of an arithmetic sequence is the product of the middle term and the number of terms



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(1) The 4^{th} term of an arithmetic sequence is 8.

(i) Find the sum of the pairs of terms given below:

(a) 3^{rd} and 5^{th}

(b) 2^{nd} and 6^{th}

(c) 1^{st} and 7^{th}

(ii) What is the sum of the 3^{rd}, 4^{th} and the 5^{th} terms?

(iii) What is the sum of the 5 terms from the 2^{nd} to the 6^{th}?

(iv) What is the sum of the 7 terms from the 1^{st} to the 7^{th}?

Answer

(i)

a) 4^{th} term = 8

3^{rd} term + 5^{th} term = 2 × 4^{th} term

= 2 × 8 = 16

b) 2^{nd} term + 6^{th} term = 2 × 4^{th} term

= 2 × 8 = 16

c) 1^{st} term + 7^{th} term= 2 × 4^{th} term

= 2 × 8 = 16

(ii) 3^{rd} term + 4^{th} term + 5^{th} term = 3 × 4^{th} term

= 3 × 8 = 24

(iii) 2^{nd} term + 3^{rd} term + 4^{th} term + 5^{th} term + 6^{th} term =5 × 4^{th} term

= 5 × 8 = 40

(iv) 1^{st}term+2^{nd}term +3^{rd}term +4^{th}term +5^{th}term+6^{th}term+ 7^{th}term

= 7 × 4^{th}term

= 7 × 8 = 56

(2) The common difference of an arithmetic sequence is 2 and the sum of the 9^{th}, 10^{th} and 11^{th} terms is 90. Calculate the first three terms of the sequence.

Answer

Common difference=2

9^{th}term +10^{th}term+11^{th}term = 90

10^{th} term = $\frac{1}{3}$ × 90 = 30

1^{st}term = 30-(9 × Common difference)

= 30– (9 ×2)= 30–18=12

2^{nd}term = 12+2=14,

3^{rd}term = 14+2=16

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(1) Write three arithmetic sequences with the sum of the first 7 terms as 70.

Answer

Sum of first 7 terms = 70

∴ Fourth term = $\frac{70}{7}$ = 10

Arithmetic sequences are:

7, 8, 9, 10, 11, 12, 13...

4, 6, 8, 10, 12, 14, 16...

-2, 2, 6, 10, 14, 18, 22...

(2) The sum of the first 3 terms of an arithmetic sequence is 30 and the sum of the first 7 terms is 140.

(i) What is the 2nd term of the sequence?

(ii) What is the 4th term of the sequence?

(iii) What are the first three terms of the sequence?

Answer

i) Sum of first 3 terms = 30

∴ Second term = $\frac{30}{3}$ = 10

ii) Sum of first 7 terms =140

∴ Fourth term = $\frac{140}{7}$ = 20

iii) Common difference = $\frac{20-10}{4-2}$ = $\frac{10}{2}$ = 5

First term = 10–5 = 5

Third term = 10 + 5 = 15

The first three terms of the sequence : 5, 10, 15

(3) The sum of the first five terms of an arithmetic sequence is 150, and the sum of the first ten terms is 550

(i) What is the third term of the sequence?

(ii) What is the eighth term of the sequence?

(iii) Write the first three terms of the sequence

Answer

i) **Sum of first 5 terms = 150**

∴ Third term = $\frac{150}{5}$ = 30

ii) **Sum of first 10 terms = 550**

The sum of the terms from the 6^{th} term to 10^{th} term =550−150=400

∴ Eighth term = $\frac{400}{5}$ = 80 **(6^{th} ,7^{th} , 8^{th} , 9^{th} ,10^{th})**

iii) **Common difference = $\frac{80-30}{8-3}$ = $\frac{50}{5}$ = 10**

Second term = 30−10=20

First term = 20−10 = 10

The first three terms of the sequence : 10, 20, 30.

(4) The sum of the 11^{th} and 21^{st} terms of an arithmetic sequence is 80. What is the 16^{th} term?

Answer

There are 11 terms from the 11^{th} term to the 21^{st} term.

16^{th} term is the middle term

(11^{th} ,12^{th} , 13^{th} , 14^{th} ,15^{th} , 16^{th} ,17^{th} , 18^{th} , 19^{th} ,20^{th}, 21^{st})

11^{th} + 21^{st} = 80

∴ 16^{th} term = $\frac{80}{2}$ = 40

(5) The angles of a pentagon are in arithmetic sequence

(i) If the angles are written according to their magnitude, what would be the third angle?

(ii) If the smallest angle is 40°, what are the other angles?

(iii) Can the smallest angle be 36°?

Answer

Sum of inner angle of polygon = (n-2) × 180°

i) **Sum of 5 angles of pentagon** $= (5-2) \times 180°$

$= 3 \times 180° = 540°$

The angles of a pentagon are in arithmetic sequence

∴ Third angle $= \frac{540}{5} = 108°$

ii) **Common difference** $= \frac{108-40}{3-1} = \frac{68}{2} = 34°$

The smallest angle is 40°.

∴ The 5 angles are 40°, 74° ,108° ,142°,176°

iii) **If the smallest angle is 36°,**

Common difference $= \frac{108-36}{3-1} = \frac{72}{2} = 36°$

∴ The 5 angles are 36°, 72°,108°, 144°, 180°

But the inner angle of a polygon cannot be 180°.

So the smallest angle cannot be 36°.

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In an arithmetic sequence, if one position is increased and another position is decreased by the same amount, the sum of the terms at these positions do not change

In an arithmetic sequence, if the sum of two positions is equal to the sum of other two positions, then the sum of the terms at each pair is the same

(1) Write four arithmetic sequence with sum of the first four terms 100

Answer

i) **Sum of first 4 terms = 100**

1^{st} term + 4^{th} term = $\frac{100}{2}$ = 50

2^{nd} term + 3^{rd} term = $\frac{100}{2}$ = 50

Any 2 numbers whose sum is 50 can be taken as the 2^{nd} and 3^{rd} terms.

Arithmetic sequences are:

10, 20, 30, 40, ...

7, 19, 31, 43, ...

4, 18, 32, 46, ...

16, 22, 28, 34, ...

(2) The 1^{st} term of an arithmetic sequence is 5 and the sum of the first 6 terms is 105. Calculate the first six terms of the sequence.

Answer

1^{st} term = 5

The sum of first 6 terms = 105

1^{st} term + 6^{th} term = $\frac{105}{3}$ = 35

6^{th} term = 35–5= 30

Common difference = $\frac{30-5}{6-1}$ = $\frac{25}{5}$ = 5

The first 6 terms of the sequence : 5, 10, 15, 20, 25, 30.

(3) The sum of the 7^{th} and the 8^{th} terms of an arithmetic sequence is 50. Calculate the sum of the first 14 terms.

Answer

If we take pairs of first 14 terms of the arithmetic sequence, we got 7 pairs.
The sum of each pairs is 50.
Sum of first 14 terms = 7 × 50 = 350

(1^{st} term +14^{th} term
2^{nd} term +13^{th} term
3^{rd} term +12^{th} term
4^{th} term + 11^{th} term
5^{th} term + 10^{th} term
6^{th} term + 9^{th} term
7^{th} term + 8^{th} term)

(4) Write the first three terms of each of the arithmetic sequences given below:

(i) The 1st term is 30 and the sum of the first three terms is 300

(ii) The 1st term is 30 and the sum of the first four terms is 300

(iii) The 1st term is 30 and the sum of the first five terms is 300

(iv) The 1st term is 30 and the sum of the first six terms is 300

Answer

i) 1^{st} **term = 30**

Sum of first 3 terms = 300

2^{nd} **term** $= \frac{300}{3} = 100$

Common difference = 100-30= 70

3^{rd} **term = 100 + 70 = 170**

The first 3 terms of the sequence : 30, 100, 170.

ii) 1^{st} **term = 30**

Sum of first 4 terms = 300

1^{st} **term +** 4^{th} **term** $= \frac{300}{2} = 150$

4^{th} **term = 150 – 30 = 120**

Common difference $= \frac{120-30}{4-1} = \frac{90}{3} = 30$

The first 3 terms of the sequence : 30, 60, 90.

iii) 1^{st} **term = 30**

Sum of first 5 terms = 300

3^{rd} **term** $= \frac{300}{5} = 60$

Common difference $= \frac{60-30}{3-1} = \frac{30}{2} = 15$

The first 3 terms of the sequence : 30, 45, 60.

iv) 1^{st} **term = 30**

Sum of first 6 terms = 300

1^{st} **term +** 6^{th} **term** $= \frac{300}{3} = 100$

6^{th} **term = 100-30=70**

Common difference $= \frac{70-30}{6-1} = \frac{40}{5} = 8$

The first 3 terms of the sequence : 30, 38, 46.